4.4

Find the value of c guaranteed by the Mean Value Theorem for Integrals for the function over the indicated interval.

48. $f(x) = \cos x$; $[-\pi/3, \pi/3]$

$\exists c \in [-\pi/3, \pi/3]$ s.t.

$\int_{-\pi/3}^{\pi/3} \cos x\,dx = f(c)\cdot\left(\frac{\pi}{3} - \left(\frac{-\pi}{3}\right)\right)$

$\sin\frac{\pi}{3} - \sin\left(\frac{-\pi}{3}\right) = \cos(c)\cdot\frac{2\pi}{3}$

$\left[\frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right)\right]\cdot\frac{3}{2\pi} = \cos(c)$

$\frac{3\sqrt{3}}{2\pi} = \cos(c)$

$c = \cos^{-1}\left(\frac{3\sqrt{3}}{2\pi}\right)$

Find the average value of the function over the interval and all values of x in the interval for which the function equals its average value.

52. $f(x) = \cos x$; $[0, \pi/2]$

$\frac{1}{\pi/2}\cdot\int_0^{\pi/2} \cos x\,dx = \frac{2}{\pi}\sin\frac{\pi}{2} = \boxed{\frac{2}{\pi}}$

Find $F'(x)$.

82. $F(x) = \int_1^x \frac{t^2}{t^2+1}\,dt$

$F(x) = \int_a^{g(x)} f(t)\,dt$

$F'(x) = f(g(x))\cdot g'(x)$

$F'(x) = \frac{x^2}{x^2+1}$

$F(x) = \int_{x^2}^{3} \cos t\,dt = -\int_3^{x^2} \cos t\,dt$

$F'(x) = -(\cos x^2)\cdot 2x$

4.5

Find the indefinite integral.

20. $\int \frac{x^3}{(1+x^4)^2}\,dx$ $= \int \frac{1}{4}\frac{du}{u^2} = \int \frac{1}{4} u^{-2}\,du = -\frac{1}{4}u^{-1} + C$

$u = 1+x^4$

$du = 4x^3\,dx$

$= \boxed{-\frac{1}{4(1+x^4)} + C}$

46. $\int x \sin x^2\,dx$ $= \int \frac{1}{2}\sin u\,du$

$u = x^2$

$du = 2x\,dx$ $= \boxed{-\frac{1}{2}\cos(x^2) + C}$

60. $\int (x+1)\sqrt{2-x}\,dx$ $= -\int (2-u+1)\sqrt{u}\,du$

$u = 2-x$ $x = 2-u$

$du = -dx$

$= \int (-3u^{1/2} + u^{3/2})\,du$

$= -2u^{3/2} + \frac{2}{5}u^{5/2} + C$

$= \boxed{-2(2-x)^{3/2} + \frac{2}{5}(2-x)^{5/2} + C}$

Evaluate the definite integral.

72. $\int_0^2 x\sqrt[3]{4+x^2}\,dx$ $= \int_{x=0}^{2} \frac{1}{2}u^{1/3}\,du = \frac{3}{8}u^{4/3}\Big| =$

$u = 4+x^2$

$du = 2x\,dx$

$= \frac{3}{8}\left(\sqrt[3]{(4+x^2)}\right)^4\Big|_0^2 =$

$= \frac{3}{8}(2^4) - \frac{3}{8}(\sqrt[3]{4})^4 = \boxed{6 - \frac{3}{2}\sqrt[3]{4}}$

92. $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos x\,dx$ $= \int u^2\,du$

$u = \sin x$

$du = \cos x\,dx$ $= \frac{1}{3}(\sin x)^3\Big|_{x=-\pi/2}^{\pi/2} = \frac{1}{3}(1-(-1))$

$= \boxed{\frac{2}{3}}$

5.2

Find the indefinite integral.

$u = x^2+3$
$du = 2x\,dx$

18. $\int \frac{x^3-3x^2+4x-9}{x^2+3}dx = \int \left(x-3+\frac{x}{x^2+3}\right)dx$

$$\begin{array}{r} x-3 \\ x^2+3 \overline{\big)\, x^3-3x^2+4x-9} \\ -(x^3+3x) \\ \hline -3x^2+x-9 \\ -(-3x^2-9) \\ \hline x \end{array}$$

$= \boxed{\frac{x^2}{2}-3x+\frac{1}{2}\ln(x^2+3)+C}$

20. $\int \frac{1}{x\ln(x^3)}dx = \frac{1}{3}\int\frac{du}{u} = \boxed{\frac{1}{3}\ln\left|\ln(x^3)\right|+C}$

$u = \ln(x^3)$

$du = \frac{1}{x^3}\cdot 3x^2 dx$

$\frac{du}{3} = \frac{dx}{x}$

Solve the differential equation.

38. $\frac{dy}{dx} = \frac{2x}{x^2-9}$, (0,4)

$\int dy = \int \frac{2x}{x^2-9}dx$

$\boxed{y = \ln|x^2-9|+C}$ general solution

$4 = \ln 9 + C$

$C = 4-\ln 9$

$\boxed{y = \ln|x^2-9|+4-\ln 9}$ particular solution

Find the average value of the function over the interval.

80. $f(x) = \sec\frac{\pi x}{6}$, [0,2]

$\int \sec u\,du = \ln|\sec u+\tan u|+C$

$\frac{1}{2}\int_0^2 \sec\frac{\pi x}{6}dx = \frac{3}{\pi}\ln\left|\sec\frac{\pi}{3}+\tan\frac{\pi}{3}\right| - \frac{3}{\pi}\ln|\sec 0+\tan 0|$

$= \frac{3}{\pi}\ln|2+\sqrt{3}| - \cancel{\frac{3}{\pi}\ln|1+0|} \to 0$

$= \boxed{\frac{3}{\pi}\ln(2+\sqrt{3})}$

5.9 Inverse Trig Functions

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

Homework #3
4.4 #45-51 odd; 75-91 odd
4.5 #7-33 odd; 41-53 odd; 57-75 odd
5.2 #1-35 odd; 43-53 odd; 61, 63
5.4 #87-107 odd
5.5 #61-68 all
5.9 #1-41 odd

due Tues.

Ch 5 Review pp.405-407 #17-24, 49-56,71-72, 99-106

Take-home quiz due Wednesday

Test ~~Friday?~~ 12/~~13~~ 16

Monday