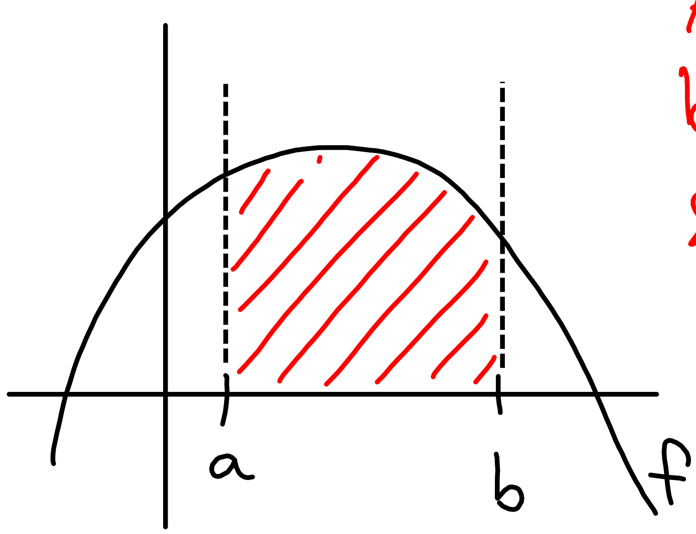
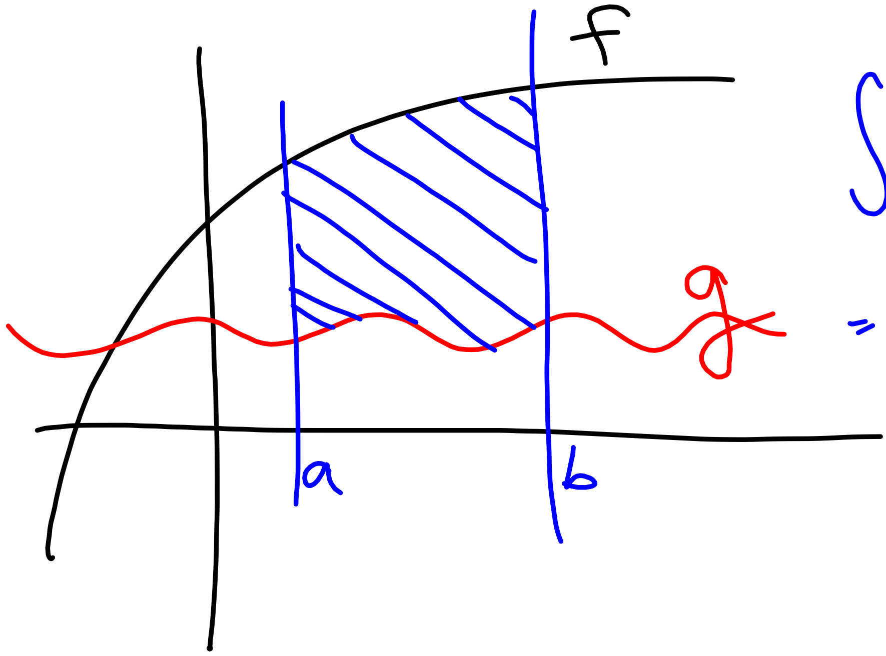


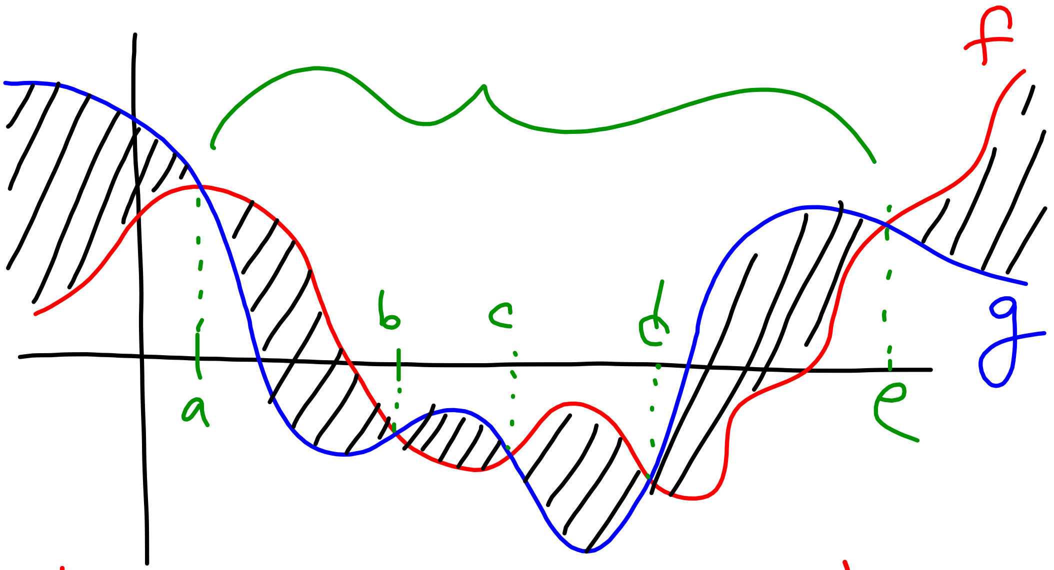
Area of region bounded by $f(x)$ & x-axis, between a and b is

$$\int_a^b f(x)\,dx$$



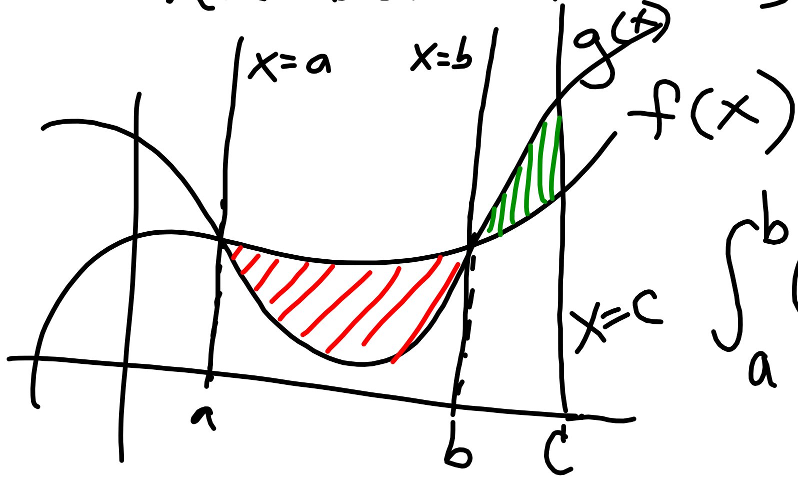
$$\int_a^b (f(x)-g(x))\,dx$$

$$= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$



$$\int_a^b (f(x)-g(x))\,dx + \int_b^c (g(x)-f(x))\,dx + \int_c^d (f(x)-g(x))\,dx +$$
$$\int_d^e (g(x)-f(x))\,dx$$

6.1 Area Between Curves

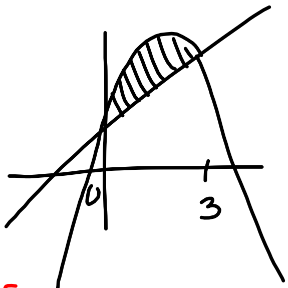


$$\int_a^b (f(x)-g(x))\,dx$$

6.1

\#18. $f(x) = -x^2 + 4x + 1$

$g(x) = x + 1$



Set $f(x) = g(x)$ & solve to find limits of integration

$-x^2 + 4x + 1 = x + 1$

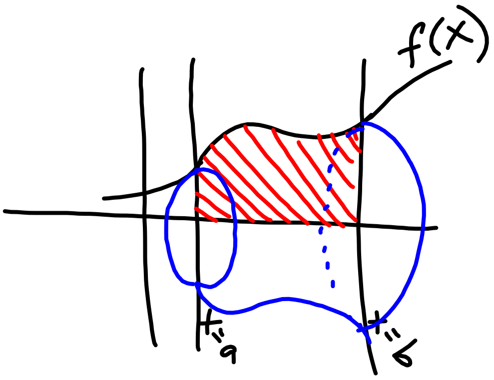
$0 = x^2 - 3x \qquad x = 0, 3 \Rightarrow \int_0^3 \left(-x^2 + 4x + 1 - (x+1)\right) dx$

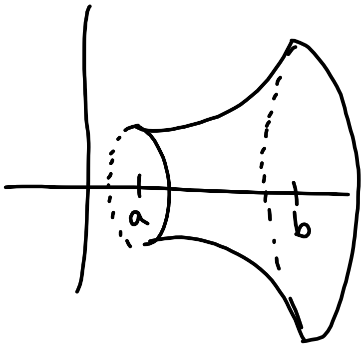
$0 = x(x-3)$

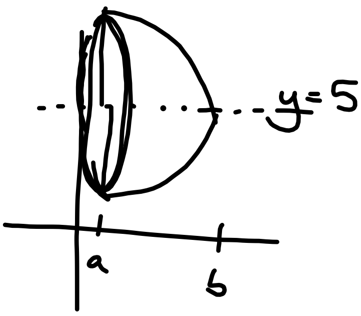
$$\int_0^3 (-x^2 + 3x)\,dx = -\frac{x^3}{3} + \frac{3}{2}x^2 \Big|_0^3 = -9 + \frac{27}{2}$$

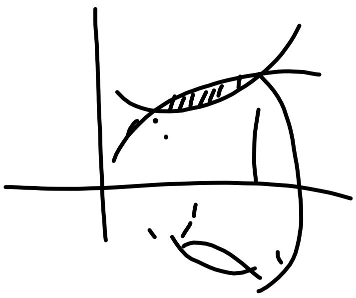
$$= \boxed{\frac{9}{2}}$$

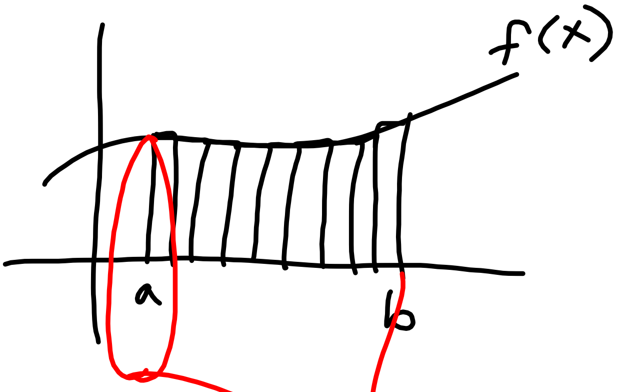
6.2 Volume of Solids of Revolution



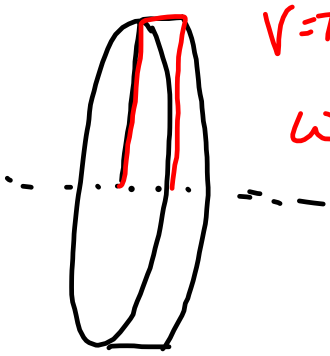








$$\int_a^b \pi \left(f(x)\right)^2 \cdot dx$$



$V = \pi r^2 \cdot \omega$

$\omega = \Delta x$
$= dx$

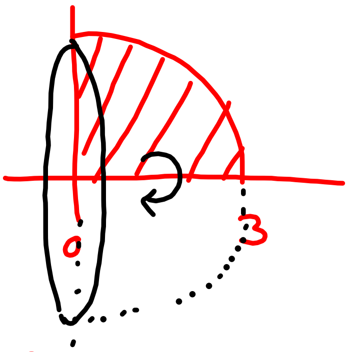
$$\int_a^b \pi r^2 \cdot dx$$

r = some function of f

6.2

4. $y = \sqrt{9 - x^2}$

$$\int_0^3 \pi \left(9 - x^2\right) dx$$



$(x-h)^2 + (y-k)^2 = r^2$

center: (h, k)

radius: r

$x^2 + y^2 = 3^2$

$y^2 = 9 - x^2$

$y = \pm\sqrt{9 - x^2}$

$$\int_0^3 \left(-\pi x^2 + 9\pi\right) dx$$

$$= \frac{-\pi}{3} x^3 + 9\pi x \Big|_0^3 = -9\pi + 27\pi$$

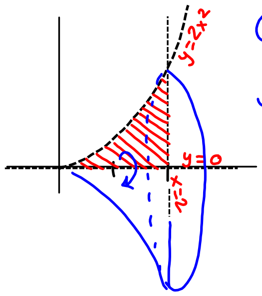
$$= \boxed{18\pi}$$

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations:

12. $y = 2x^2$, $y = 0$, $x = 2$

about the line:

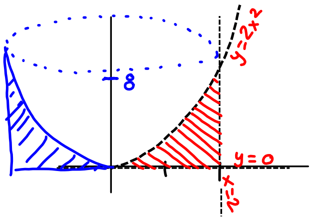
(a) y-axis, (b) x-axis, (c) $y = 8$, (d) $x = 2$



(b) x-axis

$$\int_0^2 \pi (2x^2)^2\,dx = \int_0^2 4\pi x^4\,dx =$$

$$\frac{4\pi}{5}x^5 \Big|_0^2 = \boxed{\frac{128\pi}{5}}$$



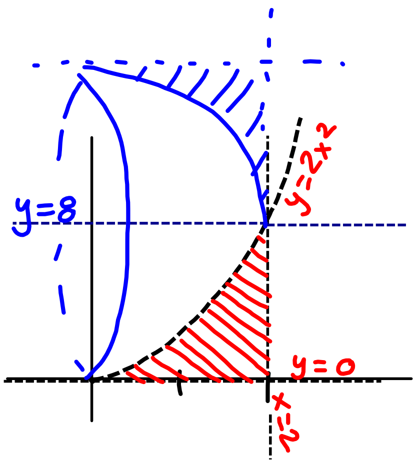
(a) y-axis

$x = \sqrt{y/2}$

Volume of solid = volume of outer cylinder minus volume of "inside of bowl"

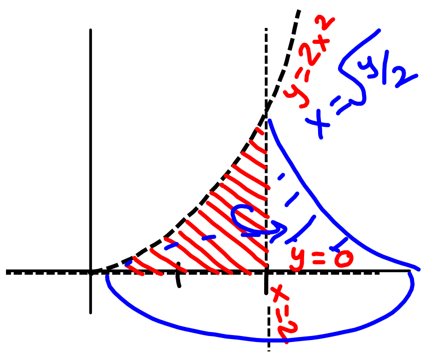
$$\pi(2)^2 \cdot 8 - \int_0^8 \pi\left(\sqrt{y/2}\right)^2 dy = 32\pi - \int_0^8 \frac{\pi}{2} y\,dy =$$

$$32\pi - \frac{\pi}{4}y^2 \Big|_0^8 = 32\pi - 16\pi = \boxed{16\pi}$$



(c) $y = 8$

$$\underbrace{\pi(8)^2 \cdot 2}_{\text{outer cylinder}} - \int_0^2 \pi\left(8 - 2x^2\right)^2 dx$$



(d) $x = 2$

$$\int_0^8 \pi\left(2 - \sqrt{y/2}\right)^2 dy$$

Homework:

- 6.1 #1-9 odd; 19, 43
- 6.2 #11, 13, 17, 19, 21, 25, 29, 35