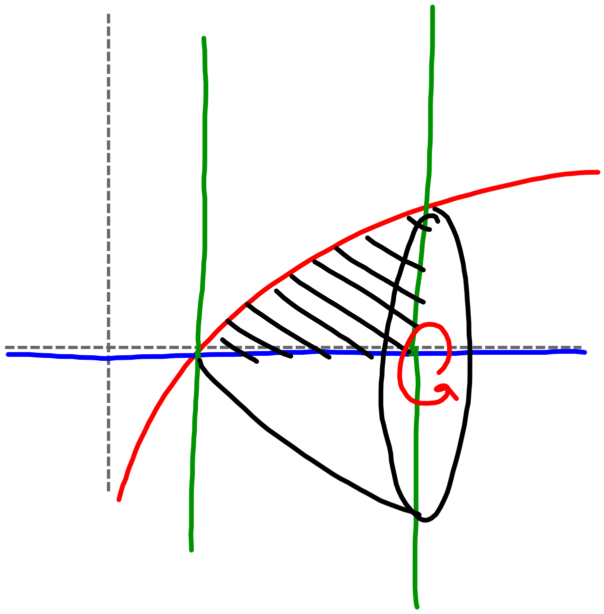
36. $y = \ln x$, $y = 0$, $x = 1$, $x = 3$

about x-axis



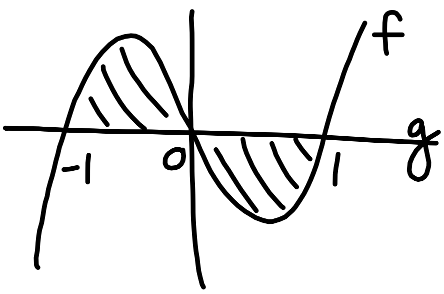
$$\int_1^3 \pi (\ln x)^2 \, dx$$

needs integration by parts!

6.1

5. $f(x) = 3(x^3 - x)$

$g(x) = 0$

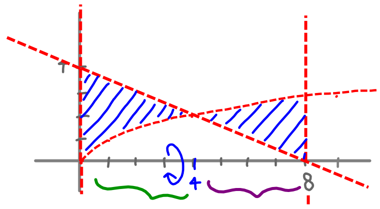



$$\int_{-1}^{0} (f - g)\,dx + \int_0^1 (g - f)\,dx$$

$$2 \cdot \int_{-1}^{0} 3(x^3 - x)\,dx$$

30. $y=\sqrt{x}$, $y=-\frac{1}{2}x+4$, $x=0$, $x=8$

about x-axis




$$\int_0^4 \pi\left(-\tfrac{1}{2}x+4\right)^2 dx - \int_0^4 \pi\left(\sqrt{x}\right)^2 dx + \int_4^8 \pi\left(\sqrt{x}\right)^2 dx - \int_4^8 \pi\left(-\tfrac{1}{2}x+4\right)^2 dx$$

$$\int_0^4\left(\frac{\pi}{4}x^2-5\pi x+16\pi\right)dx+\int_4^8\left(-\frac{\pi}{4}x^2+5\pi x-16\pi\right)dx=$$

$$=\frac{\pi}{12}x^3-\frac{5\pi}{2}x^2+16\pi x\Big|_0^4+\left(-\frac{\pi}{12}x^3+\frac{5\pi}{2}x^2-16\pi x\right)\Big|_4^8=$$

$$=\frac{\pi}{12}\cdot4^3-\frac{5\pi}{2}\cdot4^2+16\pi\cdot4-\frac{\pi}{12}\cdot8^3+\frac{5\pi}{2}\cdot8^2-16\pi\cdot8+\frac{\pi}{12}\cdot4^3-\frac{5\pi}{2}\cdot4^2+16\pi\cdot4$$

$$=\frac{16\pi}{3}-40\pi+64\pi-\frac{128\pi}{3}+160\pi-128\pi+\frac{16\pi}{3}-40\pi+64\pi$$

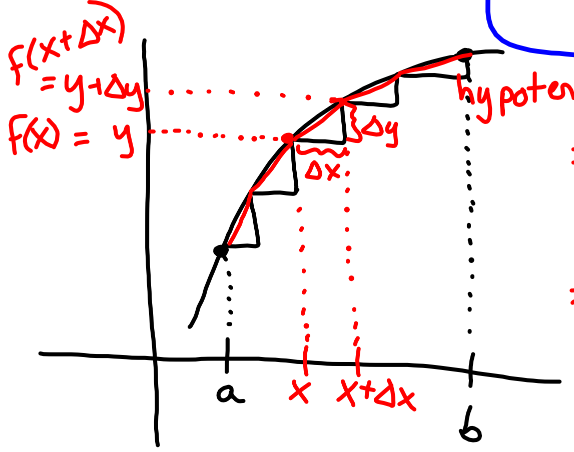
$$=\boxed{48\pi}$$

## Homework:

- 6.1 #1-9 odd; 19, 43
- 6.2 #11, 13, 17, 19, 21, 25, 29, 35

# 6.4 Arc Length & Surfaces of Revolution

The arc length $s$ of a smooth curve $f$ from $a$ to $b$ is $\boxed{s = \int_a^b \sqrt{1+[f'(x)]^2}\,dx}$




hypotenuse has length $\sqrt{(\Delta x)^2+(\Delta y)^2}$

$$= \sqrt{(\Delta x)^2\left[1+\left(\frac{\Delta y}{\Delta x}\right)^2\right]}$$

$$= \sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2}\cdot \Delta x$$

MVT $f(x+\Delta x) - f(x) = f'(c)\cdot\Delta x$

$$\frac{\Delta y}{\Delta x} = f'(c)$$

6. $y = \frac{3}{2}x^{2/3}+4$, $[1,27]$

arc length: $\int_1^{27}\sqrt{1+(y')^2}\,dx =$

$y' = x^{-1/3}$

$= \frac{1}{\sqrt[3]{x}}$

$$= \int_1^{27}\sqrt{1+\left(\frac{1}{\sqrt[3]{x}}\right)^2}\,dx =$$

$$= \int_1^{27}\sqrt{\frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x^2}}}\,dx$$

Let $u = x^{2/3}+1$

$du = \frac{2}{3}x^{-1/3}dx$

$\frac{3}{2}du = \frac{dx}{\sqrt[3]{x}}$

$$= \int_1^{27}\frac{\sqrt{\sqrt[3]{x^2}+1}}{\sqrt{(\sqrt[3]{x})^2}}\,dx = \int_1^{27}\frac{\sqrt{\sqrt[3]{x^2}+1}}{\sqrt[3]{x}}\,dx =$$

$$= \int_{x=1}^{27}\frac{3}{2}u^{1/2}\,du = u^{3/2} = \left(x^{2/3}+1\right)^{3/2}\Big|_1^{27} =$$

$$= 10^{3/2} - 2^{3/2} = \boxed{10\sqrt{10} - 2\sqrt{2}}$$