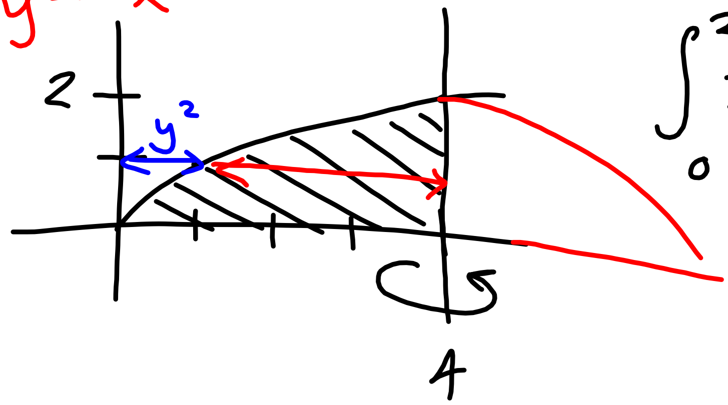
6.2

11(c)

$y=\sqrt{x}$, $y=0$, $x=4$ about $x=4$

$y^2=x$

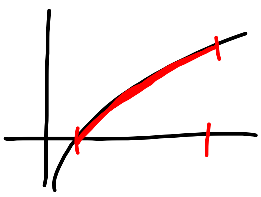


$$\int_0^2 \pi\left(4-y^2\right)^2 dy$$

6.4

The arc length s of a smooth curve f from a to b is $s=\int_a^b \sqrt{1+[f'(x)]^2}\,dx$

18. $y=\ln x$, $[1,5]$



$y'=\frac{1}{x}$

$$S=\int_1^5 \sqrt{1+\left(\tfrac{1}{x}\right)^2}\,dx$$

$$=\int_1^5 \sqrt{1+\frac{1}{x^2}}\,dx=\int_1^5 \frac{\sqrt{x^2+1}}{x}\,dx$$

$u=x^2+1$

$du=2x\,dx$

$\frac{du}{2x}=dx$

$\frac{du}{2x^2}=\frac{dx}{x}$

$$=\int_1^5 \frac{\sqrt{x^2+1}}{2x^2}\,du$$

$$=\int_1^5 \frac{\sqrt{u}}{2(u-1)}\,du$$

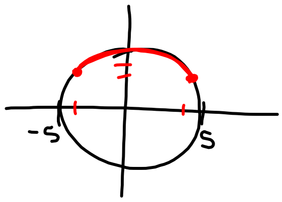
$$S = \int_1^5 \sqrt{1 + \left(\frac{1}{x}\right)^2}\, dx =$$

$$= \int_1^5 \frac{\left(1 + \frac{1}{x^2}\right) dx}{\sqrt{1 + \frac{1}{x^2}}}$$

$$= \int_1^5 \frac{1}{\sqrt{1 + \frac{1}{x^2}}} dx + \int_1^5 \frac{\frac{1}{x^2}}{\sqrt{1 + \frac{1}{x^2}}} dx$$

$$S = \int_1^5 \sqrt{1 + \left(\frac{1}{x}\right)^2}\, dx =$$

32. Find arc length from $(-3,4)$ clockwise to $(4,3)$ along the circle $x^2+y^2=25$.



$$y=\sqrt{25-x^2}$$

$$y'=\frac{1}{2}(25-x^2)^{-1/2}\cdot(-2x)$$

$$=\frac{-x}{\sqrt{25-x^2}}$$

$$\int_{-3}^{4}\sqrt{1+\left(\frac{-x}{\sqrt{25-x^2}}\right)^2}\,dx=\int_{-3}^{4}\sqrt{1+\frac{x^2}{25-x^2}}\,dx=$$

$$=\int_{-3}^{4}\sqrt{\frac{25}{25-x^2}}\,dx=\int_{-3}^{4}\frac{5\,dx}{\sqrt{25-x^2}}=$$

$$=5\arcsin\frac{x}{5}\Big|_{-3}^{4}$$

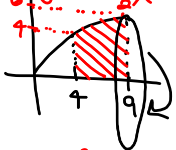
$$=5\arcsin\frac{4}{5}-5\arcsin\left(\frac{-3}{5}\right)=$$

Area of a Surface of Revolution

$$S=2\pi\int_a^b r(x)\overbrace{\sqrt{1+[f'(x)]^2}}^{\text{(arclength)}}\,dx$$

34. $y=2\sqrt{x}$, $[4,9]$ $r(x)=2\sqrt{x}$; $f'(x)=\frac{1}{\sqrt{x}}$ revolve about x-axis

$y'=2\cdot\frac{1}{2}x^{-1/2}$



$$\int_4^9 2\pi(2\sqrt{x})\sqrt{1+\left(\frac{1}{\sqrt{x}}\right)^2}\,dx$$

$$=\int_4^9 4\pi\sqrt{x}\sqrt{1+\frac{1}{x}}\,dx$$

$$=\int_4^9 4\pi\sqrt{x+1}\,dx$$

$u=x+1$

$du=dx$

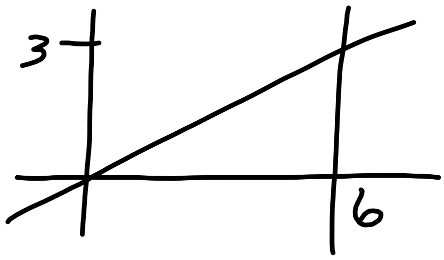
$$\int_{x=4}^{9}4\pi u^{1/2}\,du$$

$$=\frac{8\pi}{3}u^{3/2}$$

$$=\frac{8\pi}{3}(x+1)^{3/2}\Big|_{x=4}^{9}=$$

$$=\boxed{\frac{8\pi}{3}(10)^{3/2}-\frac{8\pi}{3}(5)^{3/2}}$$

36. $y = \frac{x}{2}$, $[0, 6]$ about x-axis



$$\int_0^6 2\pi\left(\frac{x}{2}\right)\sqrt{\underset{\frac{4}{4}}{1}+\underset{\frac{1}{4}}{\left(\frac{1}{2}\right)^2}}\,dx$$

$V_{cone} = \frac{1}{3}\pi r^2 h$

$$= \frac{\pi\sqrt{5}}{2}\int_0^6 x\,dx = \frac{\pi\sqrt{5}}{4}x^2\Big|_0^6$$

$$= \boxed{9\pi\sqrt{5}}$$

<u>Homework</u>:

- 6.1 #1-9 odd; 19, 43
- 6.2 #11, 13, 17, 19, 21, 25, 29, 35
- 6.4 #5, 7, 13, 33, 35

7.1 Basic Integration Rules

$$\int \frac{4}{x^2+9}\,dx$$

$$= \boxed{\frac{4}{3}\arctan\frac{x}{3} + C}$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} + C$$

$$\int \frac{4x}{x^2+9}\,dx$$

$u = x^2+9$

$du = 2x\,dx$

$2du = 4x\,dx$

$$\int \frac{2du}{u} = 2\ln|u| + c$$

$$= \boxed{2\ln(x^2+9) + C}$$

$$\int \frac{4x^2}{x^2+9}\,dx$$

. . .