Test #2

1. Find the value(s) of c guaranteed by the Mean Value Theorem for Integrals for the function over the indicated interval.

$f(x) = 1 + x^2$, $[-1,2]$

Mean Value Theorem for Integrals:
If a function f is continuous on the closed interval $[a,b]$, then there exists a number c in the closed interval $[a,b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b-a)$$

$$\int_{-1}^{2} (1+x^2)\,dx = (1+c^2)(2-(-1))$$

$$x + \tfrac{1}{3}x^3 \Big|_{-1}^{2} = (1+c^2)(3)$$

$$\boxed{c = \pm 1}$$

2. Find the average value of the function over the interval.

$f(x) = 4 - x^2$, $[-2,2]$

Average value of a function on an interval:
If f is integrable on the closed interval $[a,b]$, then the average value of f on the interval is

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\text{avg} = \frac{1}{2-(-2)}\int_{-2}^{2} (4-x^2)\,dx = \ldots = \boxed{\frac{8}{3}}$$

3. Find $F'(x)$. Test #2

$$F(x) = \int_{x^2}^{x+2} \sec^2 t\,dt = \int_{x^2}^{a} + \int_a^{x+2} = -\int_a^{x^2} \sec^2 t\,dt + \int_a^{x+2} \sec^2 t\,dt$$

The Second Fundamental Theorem of Calculus:
If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

$F'(x) =$

$$\boxed{-\sec^2(x^2)\cdot 2x + \sec^2(x+2)}$$

$$\frac{d}{dx}\left[\int_a^{g(x)} f(t)\,dt\right] = f(g(x))\cdot g'(x)$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx \qquad \int_a^b = \int_a^c + \int_c^b$$

4. Evaluate the definite integral.

$$\int_1^8 \frac{x-1}{\sqrt[3]{x^2}}\,dx = \int_1^8 \frac{x-1}{x^{2/3}}\,dx = \int_1^8 \left(\frac{x^{3/3}}{x^{2/3}} - \frac{1}{x^{2/3}}\right)dx = \int_1^8 (x^{1/3} - x^{-2/3})$$

$$= \frac{3}{4}x^{4/3} - 3x^{1/3}\Big|_1^8 = \ldots = \boxed{\frac{33}{4}}$$

Test #2

Find the indefinite integral.

5. $\int \frac{2x}{\sqrt{1-x^2}}dx$ $= \int \frac{-du}{\sqrt{u}} = \int -u^{-1/2}du = \ldots =$

$u = 1-x^2$

$du = -2x\,dx$

$-du = 2x\,dx$

$= \boxed{-2\sqrt{1-x^2} + C}$

6. $\int \frac{\sec^2 x}{1+\tan^2 x}dx$ $= \int \frac{du}{1+u^2}$

$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$

$u = \tan x$

$du = \sec^2 x\,dx$

$= \arctan u + C$

$= \arctan(\tan x) + C = \boxed{x + C}$

7. $\int \frac{2^{-1/t}}{t^2}dt$ $= \int 2^u du = \frac{2^u}{\ln 2} + C$

$\int a^x dx = \frac{a^x}{\ln a} + c$

Test #2

$u = -1/t = -t^{-1}$

$du = t^{-2}dt$

$du = \frac{dt}{t^2}$

$= \boxed{\frac{2^{-1/t}}{\ln 2} + C}$

8. $\int \frac{e^x}{e^x-1}dx$ $= \int \frac{du}{u} = \ln|u| + C$

$\int \frac{du}{u} = \ln|u| + c$

$u = e^x - 1$

$du = e^x dx$

$= \boxed{\ln|e^x - 1| + C}$

Test #2

9. $\int \frac{\ln\sqrt{x}}{x}\,dx = \int \frac{\ln x^{1/2}}{x}\,dx$

$\log a^p = p \cdot \log a$

$= \int \frac{\frac{1}{2}\ln x}{x}\,dx = \int \frac{1}{2} u\,du = \frac{1}{2}\cdot\frac{u^2}{2} =$

$u = \ln x$

$du = \frac{dx}{x}$

$= \frac{1}{4}u^2 + C = \boxed{\frac{1}{4}(\ln x)^2 + C}$

10. $\int \frac{1}{\sin 3x}\,dx = \int \csc 3x\,dx$

$\int \csc u\,du = -\ln|\csc u + \cot u| + c$

$u = 3x$

$du = 3dx$

$\frac{1}{3}du = dx$

$= \int \frac{1}{3}\csc u\,du = \boxed{-\frac{1}{3}\ln|\csc 3x + \cot 3x| + C}$

Bonus:

Test #2

A. Find a function f such that $f(1) = 0$ and $f'(x) = \frac{2^x}{x}$.

$\boxed{f(x) = \int_{a=1}^{x} \frac{2^t}{t}\,dt}$

$\int_a^1 \frac{2^t}{t}\,dt = 0$

$f'(x) = \frac{2^x}{x}$ ✓

$\boxed{a = 1}$

B. Express the limit as a definite integral and evaluate. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5}$

$\lim_{n\to\infty} \sum_{i=1}^{n} \left(0 + \frac{i}{n}\right)^4 \frac{1}{n} = \int_0^1 x^4\,dx = \ldots = \boxed{\frac{1}{5}}$

Test #2

C. Find the interval on which the curve $f(x) = \int_0^x \frac{1}{1+t+t^2}\,dt$ is concave upward.

$$f'(x) = \frac{1}{1+x+x^2}$$

$$f''(x) = \frac{(1+x+x^2)\cdot 0 - 1(1+x+x^2)'}{(1+x+x^2)^2}$$

$$= \frac{-(1+2x)}{(1+x+x^2)^2}$$

$f''(-1)$ $f''(0)$

$+$ $-\frac{1}{2}$ $-$

$$\boxed{\left(-\infty, -\tfrac{1}{2}\right)}$$

$\underline{6.2}$

#13 volume

$y = x^2$; $y = 4x - x^2$; @ $y = 6$

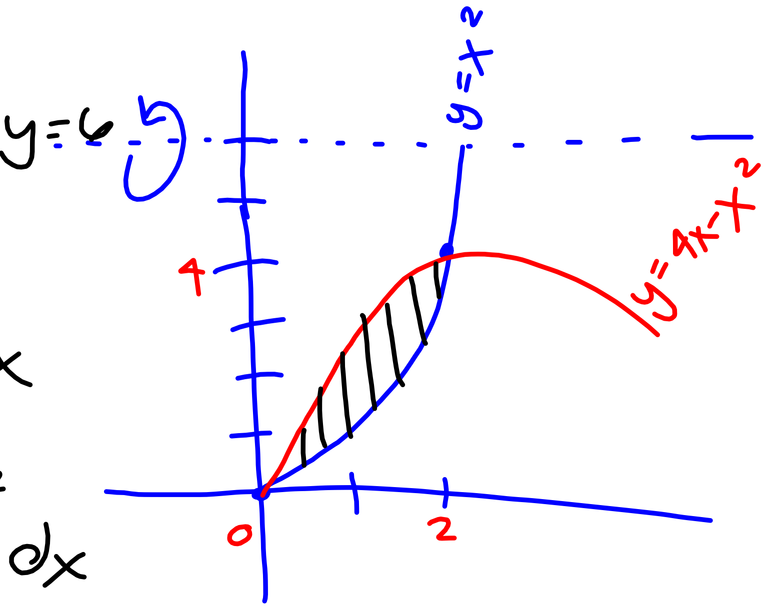
$$x^2 = 4x - x^2$$

$$2x^2 - 4x = 0$$

$$2x(x-2) = 0$$

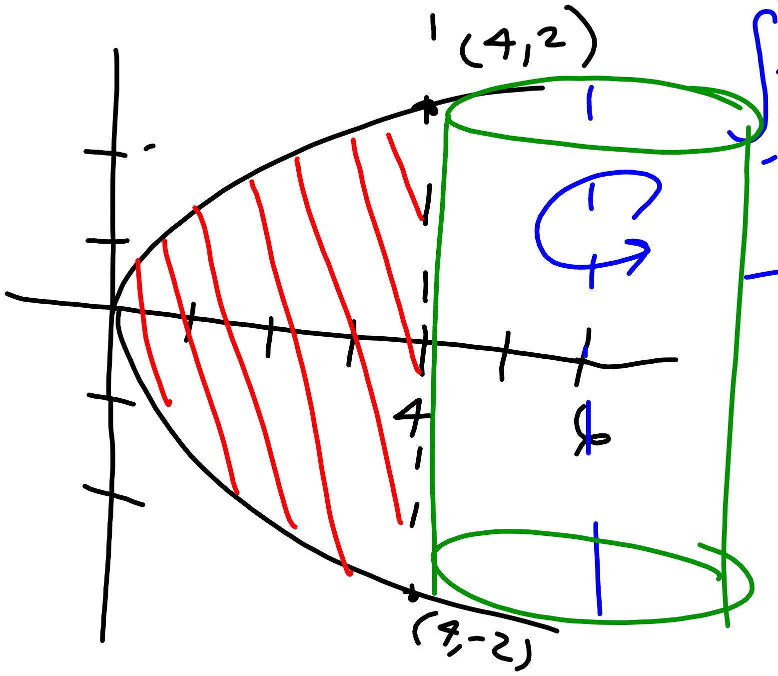
$$\int_0^2 \pi(6 - x^2)^2\,dx$$

$$\int_0^2 \pi\left(6 - (4x - x^2)\right)^2 dx$$



6.2

#21 $x=y^2$; $x=4$; @ $x=6$



$$\int_{-2}^{2}\pi\left(6-y^2\right)^2 dy - \pi(2)^2\cdot 4$$

7.1 Basic Integration Rules

$$\int \frac{4}{x^2+9}\,dx = \boxed{\frac{4}{3}\arctan\frac{x}{3}+C}$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a}+C$$

$$\int \frac{4x}{x^2+9}\,dx$$

$u=x^2+9$

$du=2x\,dx$

$2du=4x\,dx$

$$\int \frac{2du}{u} = 2\ln|u|+c = \boxed{2\ln\left(x^2+9\right)+C}$$

$$\int \frac{4x^2}{x^2+9}\,dx$$

$$\begin{array}{r} 4 \\ x^2+9\,\big)\,4x^2 \\ -(4x^2+36) \\ \hline -36 \end{array}$$

$$= \int\left(4-\frac{36}{x^2+9}\right)dx$$

$$= 4\cdot x - 36\cdot\frac{1}{3}\arctan\frac{x}{3}+C$$

$$= \boxed{4x-12\arctan\frac{x}{3}+C}$$

$$\int \frac{1}{1+e^x}dx = \int \frac{(1+e^x)-(e^x)}{1+e^x}dx$$

$$= \int \left(1 - \frac{e^x}{1+e^x}\right)dx$$

$$= x - \int \frac{e^x dx}{1+e^x} = x - \int \frac{du}{u} =$$

let $u = 1+e^x$

$du = e^x dx$

$$= x - \ln|u| + C$$

$$= \boxed{x - \ln|1+e^x| + C}$$

Homework:

- 6.1 #1-9 odd; 19, 43
- 6.2 #11, 13, 17, 19, 21, 25, 29, 35
- 6.4 #5, 7, 13, 33, 35

- 7.1 #5-53 odd
- 7.2 #1-35odd