$$\int \tan^2 2x\,dx$$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$u = 2x$

$\frac{du}{2} = \frac{2dx}{2}$

$\frac{\sin^2 x + \cos^2 x = 1}{\cos^2 x}$

$= \int \frac{1}{2}(\sec^2 u - 1)\,du$

$\tan^2 x + 1 = \sec^2 x$

$= \int \frac{1}{2}\sec^2 u\,du - \int \frac{1}{2}\,du = \frac{1}{2}\tan u - \frac{1}{2}u + C$

$= \boxed{\frac{1}{2}\tan 2x - x + C}$

$$\int \cot x \ln(\sin x)\,dx = \int u\,du = \frac{1}{2}u^2 + C$$

$u = \ln(\sin x)$

$du = \frac{1}{\sin x}\cdot \cos x\,dx$

$du = \cot x\,dx$

$= \boxed{\frac{1}{2}\left(\ln(\sin x)\right)^2 + C}$

# 7.2 Integration by Parts

$$\frac{d}{dx}[uv] = u \cdot \frac{d}{dx}[v] + v \cdot \frac{d}{dx}[u]$$

$$= uv' + vu'$$

Integrating both sides w.r.t. x yields:

$$uv = \int uv'dx + \int vu'dx$$

$$uv = \int udv + \int vdu$$

Rearranging yields:

$$\boxed{\int udv = uv - \int vdu}$$

$$\int xe^x dx$$

$$\boxed{\int udv = uv - \int vdu}$$

$u = x \qquad dv = e^x dx$

$du = dx \qquad v = \int e^x dx$

$v = e^x$

$$\int xe^x dx = xe^x - \int e^x dx = \boxed{xe^x - e^x + C}$$

6. $\int x^2 e^{2x}\,dx = \frac{1}{2}x^2e^{2x} - \int xe^{2x}\,dx$

$u = x^2$ $\quad dv = e^{2x}dx$

$du = 2x\,dx$ $\quad V = \int e^{2x}dx$

Let $m = 2x$, $\frac{dm}{2} = \frac{2\,dx}{2}$

$V = \frac{1}{2}e^{2x}$

$u = x$ ; $dv = e^{2x}dx$

$du = dx$ ; $V = \frac{1}{2}e^{2x}$

$\frac{1}{2}x^2e^{2x} - \left(\frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,dx\right)$

$\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$

16. $\int x^4 \ln x\,dx = \frac{1}{5}x^5\ln x - \int \frac{1}{5}x^4\,dx$

$u = \ln x$ $\quad dv = x^4dx$

$du = \frac{dx}{x}$ $\quad V = \frac{1}{5}x^5$

$= \frac{1}{5}x^5\ln x - \frac{1}{25}x^5 + C$

$\int \ln x\,dx = x\ln x - \int dx =$

$u = \ln x$ $\quad dv = dx$

$du = \frac{dx}{x}$ $\quad v = x$

$= x\ln x - x + C$

34. $\int 4\arccos x\,dx$

$u = \arccos x \qquad dv = 4\,dx$

$du = \frac{-1}{\sqrt{1-x^2}}\,dx \qquad v = 4x$

$= 4x\arccos x - \int \frac{-4x\,dx}{\sqrt{1-x^2}}$

$m = 1-x^2$

$dm = -2x\,dx$

$2dm = -4x\,dx$

$\int \frac{2\,dm}{\sqrt{m}} = \int 2m^{-1/2}\,dm$

$= 4m^{1/2}$

$= 4\sqrt{1-x^2}$

$= 4x\arccos x - 4\sqrt{1-x^2} + C$

30. $\int x^2\cos x\,dx = x^2\sin x - \int 2x\sin x\,dx$

$u = x^2 \qquad dv = \cos x\,dx$

$du = 2x\,dx \qquad v = \sin x$

$u = 2x \qquad dv = \sin x\,dx$

$du = 2dx \qquad v = -\cos x$

$= x^2\sin x + \left(+2x\cos x - \int +2\cos x\,dx\right)$

$= x^2\sin x + 2x\cos x - 2\sin x + C$

36. $\int e^x \cos 2x\,dx$

$u = \cos 2x \qquad dv = e^x dx$

$du = -2\sin 2x\,dx \qquad v = e^x$

$\int e^x \cos 2x\,dx = e^x \cos 2x + \int +2e^x \sin 2x\,dx$

$u = 2\sin 2x \qquad dv = e^x dx$

$du = 4\cos 2x\,dx \qquad v = e^x$

$\int e^x \cos 2x\,dx = e^x \cos 2x + 2e^x \sin 2x - 4\int e^x \cos 2x\,dx$

$5\int e^x \cos 2x\,dx = e^x \cos 2x + 2e^x \sin 2x$

$$\int e^x \cos 2x\,dx = \frac{e^x \cos 2x + 2e^x \sin 2x}{5} + C$$

## 7.2

8. $\int \ln 3x\,dx = x\ln 3x - \int dx$

$u = \ln 3x \qquad dv = dx$

$du = \frac{1}{3x}\cdot 3dx \qquad v = x$

$= \frac{dx}{x}$

$= x\ln 3x - x + C$

14. $\int \frac{e^{1/t}}{t^2}\,dt = \int -e^u\,du = -e^u + C$

$u = \frac{1}{t} = t^{-1}$

$du = -1t^{-2}\,dt$

$du = -\frac{1}{t^2}\,dt$

$= \boxed{-e^{1/t} + C}$

48. $\int_0^1 x^2 e^x\,dx = x^2e^x - \int 2xe^x\,dx$

$u = x^2 \quad dv = e^x dx$

$du = 2x\,dx \quad v = e^x$

$u = 2x \quad dv = e^x dx$

$du = 2dx \quad v = e^x$

$= x^2e^x - \left(2xe^x - \int 2e^x dx\right) =$

$= x^2e^x - 2xe^x + 2e^x \Big|_0^1 =$

$= e - 2e + 2e - (2) = \boxed{e-2}$

28. $\int x \sin x \, dx = -x\cos x + \int \cos x \, dx$

$u = x \quad dv = \sin x \, dx$

$du = dx \quad v = -\cos x$

$= -x\cos x + \sin x + C$

Homework:

- 6.1 #1-9 odd; 19, 43
- 6.2 #11, 13, 17, 19, 21, 25, 29, 35
- 6.4 #5, 7, 13, 33, 35

- 7.1 #5-53 odd
- 7.2 #1-35odd

# 7.3 Trigonometric Integrals

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

4. $\int \cos^3 x \sin^4 x \, dx$