67. $\int \frac{\ln x}{x}\,dx$

$u = \ln x$

$du = \frac{dx}{x}$

$\int u\,du$

$= \frac{u^2}{2} + C$

$= \boxed{\frac{(\ln x)^2}{2} + C}$

68. $\int x \ln x\,dx$

$u = \ln x$ $\quad dv = x\,dx$

$du = \frac{dx}{x}$ $\quad v = \frac{x^2}{2}$

$= \frac{x^2}{2}\ln x - \int \frac{1}{2} x\,dx$

$= \boxed{\frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C}$

58. $\int_0^{\pi/4} x \sec^2 x\,dx$

$u = x$ $\quad dv = \sec^2 x\,dx$

$du = dx$ $\quad v = \tan x$

$\int -\frac{du}{u}$

$\shortparallel$

$x\tan x - \int \tan x\,dx = x\tan x - \int \frac{\sin x\,dx}{\cos x}$

$u = \cos x$

$du = -\sin x\,dx$

$= x\tan x + \ln|\cos x| \Big|_0^{\pi/4}$

$\frac{1}{\sqrt{2}} = 2^{-1/2}$

$= \boxed{\frac{\pi}{4} - \frac{1}{2}\ln 2}$

Solve the differential equation.

40. $\dfrac{dy}{dx} = x^2\sqrt{x-1}$

separation of variables

$\int dy = \int x^2\sqrt{x-1}\cdot dx$

$u = x-1 \quad du = dx$

$u+1 = x$

$u^2+2u+1 = x^2$

$y = \int (u^2+2u+1)\cdot u^{1/2}\,du$

$y = \int (u^{5/2}+2u^{3/2}+u^{1/2})\,du$

$y = \frac{2}{7}u^{7/2} + \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C$

General Solution:

$$\boxed{y = \frac{2}{7}(x-1)^{7/2} + \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C}$$

7.3 Trigonometric Integrals

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$\sin 4x = \sin 2(2x)$

$= 2\sin 2x\cos 2x$

$\sin 38x = \sin 2(19x)$

$= 2\sin 19x\cos 19x$

4. $\int \cos^3 x \sin^4 x\,dx$

$= \int \underline{\cos^2 x}\cdot\sin^4 x\cdot\cos x\,dx$

$= \int (1-\sin^2 x)\sin^4 x\cdot\cos x\,dx$

$= \int (\sin^4 x - \sin^6 x)\underline{\cos x\,dx}$

$u = \sin x$

$du = \cos x\,dx$

$= \int (u^4 - u^6)\,du = \frac{1}{5}u^5 - \frac{1}{7}u^7 + C$

$$= \boxed{\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C}$$

12. $\int \sin^2 2x\,dx = \int (\sin 2x)^2\,dx =$

$\int (1-\cos^2 2x)\,dx$

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

$= \int (2\sin x\cos x)^2\,dx$

$\boxed{= \int 4\underbrace{\sin^2 x}_{}\cos^2 x\,dx}$ $(1-\cos^2 x)$

$\cancel{u = \sin^2 x = (\sin x)^2}$

$\cancel{du = 2\sin x\cos x}$

$\cancel{u = \sin x}$

$\cancel{du = \cos x}\,dx$

$= \int 4(1-\cos^2 x)\cos^2 x\,dx = \int (4\cos^2 x - 4\cos^4 x)\,dx$

$\int 4\left(\dfrac{1-\cos 2x}{2}\right)\cos^2 x\,dx$

$\cos 2x = 2\cos^2 x - 1$

$\dfrac{\cos 2x + 1}{2} = \cos^2 x$

$= \int (4\cos^2 x - 4\cos^4 x)\,dx$

$= \int 4\cos^2 x\,dx - \int 4\cos^4 x\,dx$

$= \int 4\left(\dfrac{\cos 2x + 1}{2}\right)dx - \int$

$= \int (2\cos 2x + 2)\,dx - \int$

$= \sin 2x + 2x - \int 4\cos^4 x\,dx$

$- \int \cancel{4}\left(\dfrac{\cos 2x + 1}{\cancel{2}}\right)\left(\dfrac{\cos 2x + 1}{\cancel{2}}\right)dx$

$- \int (\cos^2 2x + 2\cos 2x + 1)\,dx$

$= \sin 2x + 2x - \int \cos^2 2x\,dx - \int 2\cos 2x\,dx - \int dx$

$= \sin 2x + 2x$

$- \sin 2x - x + C$

12. $\int \sin^2 2x\, dx$

$\cos 2x = 1 - 2\sin^2 x$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

$\sin^2 (2x) = \dfrac{1-\cos 4x}{2}$

$= \int \left(\dfrac{1-\cos 4x}{2}\right) dx$

$= \int \frac{1}{2}\, dx - \int \frac{1}{2}\cos 4x\, dx$

$= \boxed{\frac{1}{2}x - \frac{1}{8}\sin 4x + C}$

26. $\int \tan^2 x\, dx = \int (\sec^2 x - 1)\, dx$

$= \int \sec^2 x\, dx - \int dx$

$= \boxed{\tan x - x + C}$

38. $\int \frac{\tan^2 x}{\sec^5 x}\,dx$

$$\int \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^5 x}{1}\,dx$$

$$= \int \sin^2 x \underbrace{\cos^3 x}\,dx$$

$$= \int \sin^2 x \,(1-\overset{\cos^2 x}{\sin^2 x})\cos dx$$

$$= \int(\sin^2 x - \sin^4 x)\cos x\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$$\int (u^2 - u^4)\,du$$

$$= \frac{1}{3}u^3 - \frac{1}{5}u^5 + C$$

$$= \boxed{\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C}$$

Homework:

- 6.1 #1-9 odd; 19, 43 (area between curves)
- 6.2 #11, 13, 17, 19, 21, 25, 29, 35 (volume of solid of revolution)
- 6.4 #5, 7, 13, 33, 35 (arc length and surface of revolution)

- 7.1 #5-53 odd (basic integration rules)
- 7.2 #1-35odd (integration by parts)

- 7.3 #3-15odd; 21-37odd; 47-67odd (trigonometric integrals)
- 7.4 #5-15odd; 19-43odd (trigonometric substitution)