7.4

30. $\int \frac{dx}{x\sqrt{4x^2+16}} = \int \frac{dx}{2x\sqrt{x^2+4}}$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$= \int \frac{2\sec^2\theta\, d\theta}{2(2\tan\theta)\sqrt{4\tan^2\theta+4}}$

$\sqrt{4(\tan^2\theta+1)} = \sqrt{4\sec^2\theta} = 2\sec\theta$

$= \int \frac{\sec^2\theta\, d\theta}{2\tan\theta \cdot 2\sec\theta}$

$= \int \frac{\sec\theta\, d\theta}{4\tan\theta}$

$x = 2\tan\theta$, $\frac{x}{2} = \tan\theta$, $\sqrt{x^2+2^2}$, 2, x

$= \int \frac{1}{4} \cdot \frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta}\, d\theta = \int \frac{1}{2} \cdot \frac{\cos\theta\, d\theta}{\sin 2\theta}$

$= \int \frac{1}{4}\csc\theta\, d\theta = -\frac{1}{4}\ln|\csc\theta+\cot\theta| + C$

$= -\frac{1}{4}\ln\left|\frac{\sqrt{x^2+2}}{x} + \frac{2}{x}\right| + C$

$\int \csc u\, du = -\ln|\csc u+\cot u|$

$= \ln\left|\frac{1}{\csc u+\cot u}\right|$

$v = \frac{1}{\csc u+\cot u}$

$dv = -\frac{1}{(\csc u+\cot u)^2} \cdot (-\csc u\cot u - \csc^2 u)$

$= \frac{-\csc u(\cot u+\csc u)}{(\csc u+\cot u)^2}$

$dv = \frac{-\csc u}{\csc u+\cot u}\, du$

$\int \frac{-\csc u\, du}{\frac{1}{\csc u+\cot u}} = \int \frac{dv}{v}$

$= \int \csc u\, du$

40. $\int x\arcsin x\, dx$

$u = \arcsin x$ $\quad dv = x\,dx$

$du = \frac{1\,dx}{\sqrt{1-x^2}}$ $\quad v = \frac{1}{2}x^2$

$= \frac{1}{2}x^2\arcsin x - \int \frac{x^2\,dx}{2\sqrt{1-x^2}}$

$x = \sin\theta$, $dx = \cos\theta\, d\theta$

$1-x^2 = 1-\sin^2\theta = \cos^2\theta$

$\sin\theta = \frac{x}{1}$, 1, x, $\sqrt{1-x^2}$

$= \frac{1}{2}x^2\arcsin x - \int \frac{\sin^2\theta \cdot \cos\theta\, d\theta}{2\cos\theta}$

$= \frac{1}{2}x^2\arcsin x - \int \frac{1}{2}\sin^2\theta\, d\theta$

$\cos 2\theta = 1-2\sin^2\theta$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$= \frac{1}{2}x^2\arcsin x - \int \frac{1}{4}(1-\cos 2\theta)\, d\theta$

$= \frac{1}{2}x^2\arcsin x - \frac{1}{4}\theta + \frac{1}{8}\sin 2\theta + C$

$= \frac{1}{2}x^2\arcsin x - \frac{1}{4}\arcsin x + \frac{1}{8} \cdot 2\sin\theta\cos\theta + C$

$= \frac{1}{2}x^2\arcsin x - \frac{1}{4}\arcsin x + \frac{1}{4} \cdot x \cdot \sqrt{1-x^2} + C$

Test 3

(tentatively)

Wed. 01/29

6. $\int e^x \cos x \, dx$

$u = \cos x \quad dv = e^x dx$

$du = -\sin x \, dx \quad v = e^x$

$\int = e^x \cos x + \int +e^x \sin x \, dx$

$u = \sin x \quad dv = e^x dx$

$du = \cos x \, dx \quad v = e^x$

$\int e^x \cos x \, dx = e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$

$2\int e^x \cos x \, dx = e^x \cos x + e^x \sin x$

$$\int e^x \cos x \, dx = \frac{e^x \cos x + e^x \sin x}{2} + C$$

8. $\int \sin^2 x \cos^2 x \, dx$

$= \int (1 - \cos^2 x) \cos^2 x \, dx$

$= \int \cos^2 x \, dx - \int \cos^4 x \, dx$

$\cos 2\theta = 2\cos^2 \theta - 1$

$\frac{\cos 2\theta + 1}{2} = \cos^2 \theta$

. . .

Homework:

- 6.1 #1-9 odd; 19, 43 (area between curves)
- 6.2 #11, 13, 17, 19, 21, 25, 29, 35 (volume of solid of revolution)
- 6.4 #5, 7, 13, 33, 35 (arc length and surface of revolution)

- 7.1 #5-53 odd (basic integration rules)
- 7.2 #1-35odd (integration by parts)

- 7.3 #3-15odd; 21-37odd; 47-67odd (trigonometric integrals)
- 7.4 #5-15odd; 19-43odd (trigonometric substitution)