8. $\int \sin^2 x \cos^2 x \, dx$

$= \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx$

$= \int \frac{1}{4}(1-\cos^2 2x)\, dx$

$= \int \frac{1}{4} dx - \int \frac{1}{4}\cos^2 2x \, dx$

$= \int \frac{1}{4} dx - \int \frac{1}{4} \cdot \frac{\cos 4x + 1}{2} dx$

$= \frac{1}{4}x - \int \frac{1}{8}\cos 4x - \int \frac{1}{8} dx$

$= \frac{1}{4}x - \frac{1}{32}\sin 4x - \frac{1}{8}x + C$

$= \boxed{\frac{1}{8}x - \frac{1}{32}\sin 4x + C}$

Last time:

$= \int (1-\cos^2 x)\cos^2 x \, dx$

$= \int \cos^2 x \, dx - \int \cos^4 x \, dx$

$\cos 2\theta = 2\cos^2\theta - 1$

$\frac{\cos 2\theta + 1}{2} = \cos^2\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\cos 2\theta = 2\cos^2\theta - 1$

$\frac{\cos 2\theta + 1}{2} = \cos^2\theta$

9. $\int \tan^3 x \sec^3 x \, dx$

$\int (\tan^3 x \sec x) \cdot \sec^2 x \, dx$

$\tan^2 x = \sec^2 x - 1$

$\int (\tan^2 x \sec^2 x) \sec x \tan x \, dx$

$\int (\sec^2 x - 1) \cdot \sec^2 x \cdot \sec x \tan x \, dx$

$u = \sec x$

$\int (u^4 - u^2) du = \frac{u^5}{5} - \frac{u^3}{3} + C = \boxed{\frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C}$

10. $\int \sin^3 x \cos^3 x \, dx$

$\int \underbrace{\sin^2 x}_{1-\cos^2 x} \cdot \cos^3 x \cdot \sin x \, dx$

$\int (\cos^3 x - \cos^5 x) \sin x \, dx$

$u = \cos x$

$du = -\sin x \, dx$

$\int (-u^3 + u^5) \, du$

$-\frac{\cos^4 x}{4} + \frac{\cos^6 x}{6} + C_1$

$\int \sin^3 x \cdot \underbrace{\cos^2 x}_{1-\sin^2 x} \cdot \cos x \, dx$

$\int (\sin^3 x - \sin^5 x) \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$\int (u^3 - u^5) \, du$

$\frac{\sin^4 x}{4} - \frac{\sin^6 x}{6} + C_2$

11. $\int \frac{1}{(25-x^2)^{3/2}} \, dx = \int \frac{5\cos\theta \, d\theta}{5^3 \cos^3\theta} =$

Let $x = 5\sin\theta$ $dx = 5\cos\theta \, d\theta$

$x^2 = 25\sin^2\theta$

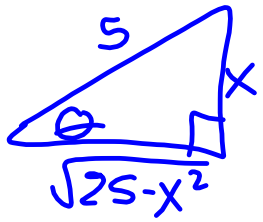
$(25-x^2)^{3/2} =$

$= (25 - 25\sin^2\theta)^{3/2}$

$= [25(1-\sin^2\theta)]^{3/2}$

$= [25\cos^2\theta]^{3/2}$

$= 5^3 \cos^3\theta$



$= \int \frac{d\theta}{25\cos^2\theta}$

$= \int \frac{1}{25} \sec^2\theta \, d\theta$

$= \frac{1}{25}\tan\theta + C$

$= \frac{x}{25\sqrt{25-x^2}} + C$

Area Between Curves

$f(x) = \sin x + 5$

$g(x) = -\sin x + 5$ $\quad 0 \le x < 5\pi$

$\sin x + 5 = -\sin x + 5$

$2\sin x = 0$

$\sin x = 0$

$x = 0, \pi, 2\pi, 3\pi, 4\pi, (5\pi)$

$\int_0^{\pi} f-g\, 2\sin x\,dx + \int_{\pi}^{2\pi} g-f\, -2\sin x\,dx + \int_{2\pi}^{3\pi} 2\sin x\,dx + \int_{3\pi}^{4\pi} -2\sin x\,dx + \int_{4\pi}^{5\pi} 2\sin x\,dx$

$6\int_0^{\pi} \sin x\,dx - 4\int_{\pi}^{2\pi} \sin x\,dx$

$-6\cos x\Big|_0^{\pi} + 4\cos x\Big|_{\pi}^{2\pi}$

$-6(-1) - [-6(1)] + 4(1) - 4(-1)$

$6 + 6 + 4 + 4$

$= 20$

Volume of solid of revolution

$f(x) = \sqrt{x}$

$y = 0$

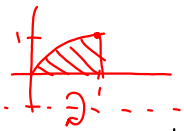
$x = 1$

$\int_a^b \pi [F(x)]^2\,dx$

a. rot. @ x-axis

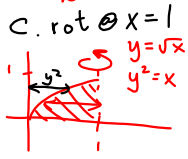
$\int_0^1 \pi(\sqrt{x})^2\,dx$

b. rot @ $y=-1$



$\int_0^1 \pi(\sqrt{x}+1)^2\,dx - \pi(1)^2 \cdot 1$

c. rot @ $x=1$



$\int_0^1 \pi(1-y^2)^2\,dy$

d. @ $x=-2$

$\pi(3)^2 \cdot 1 - \int_0^1 \pi(2+y^2)^2\,dy$

e. about y-axis

$\pi(1)^2 \cdot 1 - \int_0^1 \pi(y^2)^2\,dy$

Homework:

- 6.1 #1-9 odd; 19, 43 (area between curves)
- 6.2 #11, 13, 17, 19, 21, 25, 29, 35 (volume of solid of revolution)
- 6.4 #5, 7, 13, 33, 35 (arc length and surface of revolution)

- 7.1 #5-53 odd (basic integration rules)
- 7.2 #1-35odd (integration by parts)

- 7.3 #3-15odd; 21-37odd; 47-67odd (trigonometric integrals)
- 7.4 #5-15odd; 19-43odd (trigonometric substitution)

- **OLD TEST #3 problems**