<u>Homework for Test 3</u>:
HW #4:
- 6.1 #1-9 odd; 19, 43 (area between curves)
- 6.2 #11, 13, 17, 19, 21, 25, 29, 35 (volume of solid of revolution)
- 6.4 #5, 7, 13, 33, 35 (arc length and surface of revolution)

- 7.1 #5-53 odd (basic integration rules)
- 7.2 #1-35odd (integration by parts)

HW #5:
- 7.3 #3-15odd; 21-37odd; 47-67odd (trigonometric integrals)
- 7.4 #5-15odd; 19-43odd (trigonometric substitution)
- OLD TEST #3 problems

<u>Test 3</u> Applied part (volume, arc length, surface area) - <u>Tuesday</u>

<u>Test 3</u> Integration part - <u>Wednesday</u>

<u>7.4</u>

41. $\displaystyle\int \frac{1}{\sqrt{4x-x^2}}\,dx$

$-(x^2-4x+4)$ $+4$

$\sqrt{a^2-u^2}$

$\displaystyle\int \frac{1}{\sqrt{4-(x-2)^2}}$

$a=2$

$u=x-2$

7.5 Partial Fractions

$$\int \frac{1}{x^2-5x+6}\,dx$$

$$\frac{1}{\underset{(x-3)(x-2)}{x^2-5x+6}} = \frac{A}{x-3} + \frac{B}{x-2}$$

$$\frac{1}{x^2-5x+6} = \frac{A(x-2)+B(x-3)}{(x-3)(x-2)}$$

$$\frac{1}{x^2-5x+6} = \frac{Ax-2A+Bx-3B}{(x-3)(x-2)}$$

$$\frac{1}{x^2-5x+6} = \frac{(A+B)x+(-2A-3B)}{(x-3)(x-2)}$$

$A+B=0$

$A=-B$

$-2A-3B=1$

$-2(-B)-3B=1$

$2B-3B=1$

$-B=1$

$B=-1$

$A=1$

$$= \int\left(\frac{1}{x-3} + \frac{-1}{x-2}\right)dx$$

$$= \boxed{\ln|x-3| - \ln|x-2| + C}$$

$$\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx$$

$x(x^2+2x+1)$

$x(x+1)(x+1)$

$$\frac{5x^2+20x+6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$= \frac{A(x+1)^2 + Bx(x+1) + Cx}{x(x+1)^2} =$$

$$= \frac{Ax^2+2Ax+A+Bx^2+Bx+Cx}{x(x+1)^2} =$$

$$= \frac{(A+B)x^2+(2A+B+C)x+A}{x(x+1)^2}$$

$A+B=5$

$2A+B+C=20$

$A=6$

$B=-1$

$C=9$

$$\int\left(\frac{6}{x} + \frac{-1}{x+1} + \frac{9}{(x+1)^2}\right)dx$$

$u=x+1$

$du=dx$

$9u^{-2}du$

$$= \boxed{6\ln|x| - \ln|x+1| - \frac{9}{x+1} + C}$$

$$\int \frac{2x^3-4x-8}{(x^2-x)(x^2+4)}dx$$

$$\frac{2x^3-4x-8}{x(x-1)(x^2+4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+4}$$

$A(x-1)(x^2+4) + Bx(x^2+4) + (Cx+D)(x)(x-1)$
$(Ax-A)(x^2+4) + Bx(x^2+4) + (Cx+D)(x^2-x)$
$Ax^3+4Ax-Ax^2-4A + Bx^3+4Bx + Cx^3-Cx^2+Dx^2-Dx$
$x^3(A+B+C)+x^2(-A-C+D)+x(4A+4B-D)-4A$

$A+B+C=2$
$-A-C+D=0$
$4A+4B-D=-4$
$-4A=-8$

$A=2$
$B+C=0 \Rightarrow B=-C$
$D-C=2 \Rightarrow D=C+2$
$4B-D=-12 \Rightarrow 4(-C)-(C+2)=-12$
$-4C-C-2=-12$
$-5C=-10$
$C=2$

$B=-2$
$D=4$

$$\int\left(\frac{2}{x}+\frac{-2}{x-1}+\frac{2x+4}{x^2+4}\right)dx$$

$$=2\ln|x|-2\ln|x-1| + \int\left(\frac{2x}{x^2+4}+\frac{4}{x^2+4}\right)dx$$

$$=\boxed{2\ln|x|-2\ln|x-1|+\ln|x^2+4|+2\arctan\frac{x}{2}+C}$$

44. $$\int\frac{\sec^2 x}{\tan x(\tan x+1)}dx = \int\frac{1}{u(u+1)}du$$

$u=\tan x$
$du=\sec^2 x\,dx$

$$\frac{1}{u(u+1)} = \frac{A}{u}+\frac{B}{u+1}$$
$$=\frac{A(u+1)+Bu}{u(u+1)}$$
$$=\frac{(A+B)u+A}{u(u+1)}$$

$A+B=0$
$A=1$; $B=-1$

$$\int\left(\frac{1}{u}+\frac{-1}{u+1}\right)du$$

$$=\boxed{\ln|\tan x|-\ln|\tan x+1|+C}$$

6. $\frac{2x-1}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$

14. $\int \frac{x^3-x+3}{x^2+x-2}\,dx = \int \left[(x-1) + \frac{2x+1}{x^2+x-2}\right] dx$

$$\begin{array}{r|l} & x - 1 \\ \hline x^2+x-2 & x^3 - x + 3 \\ & -(x^3 + x^2 - 2x) \\ \hline & -x^2 + x + 3 \\ & -(-x^2 - x + 2) \\ \hline & 2x + 1 \end{array}$$

5.7 Solving Differential Equations by Separation of Variables

$$y = 5x^3 - \cos x$$

what is the differential of y?

$$dy = (15x^2 + \sin x)\,dx$$

$$y' = \frac{dy}{dx} = \frac{d}{dx}(y)$$

5.7 Ex 3 - Find the general solution.

$$(x^2+4)\frac{dy}{dx} = xy$$

$$\int \frac{dy}{y} = \int \frac{x\,dx}{x^2+4}$$

$$\ln|y| = \frac{1}{2}\ln|x^2+4| + C_1$$

$$e^{\ln|y|} = e^{\frac{1}{2}\ln|x^2+4| + C_1}$$

$\ln(x)^p = p\ln x$

$$|y| = e^{\ln\sqrt{x^2+4}}\,e^{C_1}$$

$$y = \pm e^{C_1} e^{\ln\sqrt{x^2+4}}$$

$$y = C_2 \cdot \sqrt{x^2+4}$$

Homework:
7.5 - partial fractions #15-27 odd
5.7 - separation of variables #55, 57, 59
6.5 - work #9,11,39,40