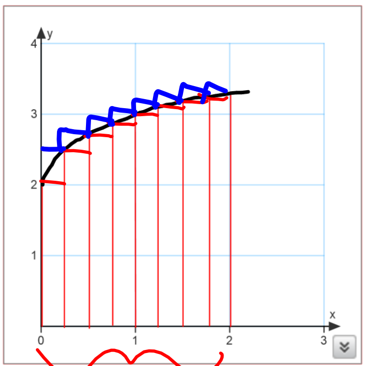
Textbook Problems:

- 3.9 #5, 9; 11-19 odd; **37**, 45, ~~49~~
- 4.1 #**3-31** odd; **35-41 odd**; **53,** 67, **75**, ~~55-61 odd; 83~~
- 4.2 #7-**23**odd; **33-41odd,45,51**, ~~27-37odd; 41,43,47,53~~
- 4.3 #7,17,37,43,~~45~~,**47**
- 4.4 #13, 15, 23, ~~31~~, **33**

$$y = \sqrt{x} + 2$$

Use upper sum and lower sum to approximate the area under the curve.



8 Intervals each of width $\frac{1}{4}$

lower sum:

$$\frac{1}{4}\cdot\left[2+\left(\sqrt{\tfrac{1}{4}}+2\right)+\left(\sqrt{\tfrac{1}{2}}+2\right)+\left(\sqrt{\tfrac{3}{4}}+2\right)+(1+2)+\left(\sqrt{\tfrac{5}{4}}+2\right)+\left(\sqrt{\tfrac{3}{2}}+2\right)+\left(\sqrt{\tfrac{7}{4}}+2\right)\right]$$

Upper sum =

$$\frac{1}{4}\left[\left(\sqrt{\tfrac{1}{4}}+2\right)+\left(\sqrt{\tfrac{1}{2}}+2\right)+\cdots+\sqrt{2}+2\right]$$

$$\lim_{n\to\infty} S(n)$$

32. $S(n) = \frac{64}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$

$$\lim_{n\to\infty} S(n) = \lim_{n\to\infty} \frac{\cancel{128}^{64}\,\cancel{n^3}}{\cancel{6}_{3}\,\cancel{n^3}} = \boxed{\frac{64}{3}}$$

rewrite without summation notation

36. $\sum_{j=1}^{n} \frac{4j+3}{n^2} = \frac{4\cdot 1+3}{n^2} + \frac{4\cdot 2+3}{n^2} + \frac{4\cdot 3+3}{n^2} \dots$

$$= \frac{1}{n^2}\sum_{j=1}^{n}(4j+3) = \frac{1}{n^2}\sum_{j=1}^{n} 4j + \frac{1}{n^2}\sum_{j=1}^{n} 3$$

$$= \frac{4}{n^2}\sum_{j=1}^{n} j + \frac{1}{n^2}\sum_{j=1}^{n} 3$$

$$= \frac{4}{n^2}\cdot\frac{n(n+1)}{2} + \frac{1}{n^2}\cdot 3n$$

44. $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left(1+\frac{2i}{n}\right)^3\left(\frac{2}{n}\right)$

$= \lim\limits_{n\to\infty}\frac{2}{n}\sum\limits_{i=1}^{n}\left(1+\frac{2i}{n}\right)^3$

$= \lim\limits_{n\to\infty}\frac{2}{n}\sum\limits_{i=1}^{n}\left(1+\frac{6i}{n}+\frac{12i^2}{n^2}+\frac{8i^3}{n^3}\right)$

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

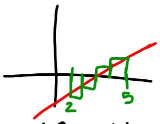
$= \lim\limits_{n\to\infty}\frac{2}{n}\left[\sum\limits_{i=1}^{n}1+\frac{6}{n}\sum\limits_{i=1}^{n}i+\frac{12}{n^2}\sum\limits_{i=1}^{n}i^2+\frac{8}{n^3}\sum\limits_{i=1}^{n}i^3\right]$

$= \lim\limits_{n\to\infty}\left[\frac{2}{n}\left(n+\frac{6}{n}\cdot\frac{n(n+1)}{2}+\frac{12}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}+\frac{8}{n^3}\cdot\frac{n^2(n+1)^2}{4}\right)\right]$

$= \lim\limits_{n\to\infty}\left[\frac{2n}{n}+\frac{2\cdot6\cdot n^2}{2n^2}+\frac{2\cdot12\cdot2n^3}{6n^3}+\frac{2\cdot8\cdot n^4}{4n^4}\right]$

$= \lim\limits_{n\to\infty}\left[2+6+8+4\right]=\boxed{20}$

48. $y=3x-4$, $[2,5]$



Area $= \lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}f(c_i)\Delta x$, $x_i \le c_i \le x_{i+1}$, $\Delta x = \frac{b-a}{n}$

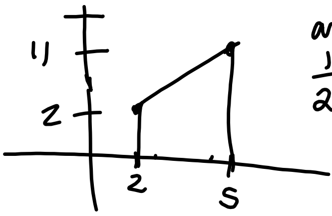
$= \lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\underbrace{\left(\frac{5-2}{n}\right)}_{\text{width of } i^{th} \text{ rectangle}}^{=\frac{3}{n}}\underbrace{\left(3\left(2+\frac{3i}{n}\right)-4\right)}_{\text{height of } i^{th} \text{ rectangle}}$

right-hand endpoint of i^{th} interval

$= \lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left[\frac{18}{n}+\frac{27i}{n^2}-\frac{12}{n}\right]=\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\left(\frac{6}{n}+\frac{27i}{n^2}\right)$

$= \lim\limits_{n\to\infty}\left[\frac{6}{n}\cdot n+\frac{27}{n^2}\cdot\frac{n(n+1)}{2}\right]=6+\frac{27}{2}=\boxed{\frac{39}{2}}$

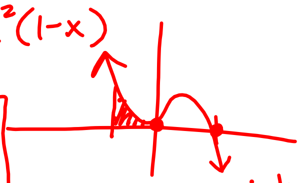
$y=3x-4$
$[2,5]$



area $= \frac{1}{2}(2+11)(3)=\frac{39}{2}$

$=\boxed{\frac{39}{2}}$

56. $y=x^2-x^3$ $\quad [-1,0]$ $\quad y=x^2(1-x)$



$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\cdot\left[\left(-1+\frac{1}{n}i\right)^2-\left(-1+\frac{1}{n}i\right)^3\right]$$

left endpoint + width of interval times index var.

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\left[\left(1-\frac{2i}{n}+\frac{i^2}{n^2}\right)-\left(-1+\frac{3i}{n}-\frac{3i^2}{n^2}+\frac{i^3}{n^3}\right)\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\left(2-\frac{5i}{n}+\frac{4i^2}{n^2}-\frac{i^3}{n^3}\right)$$

$$=\lim_{n\to\infty}\left(\frac{1}{n}2n-\frac{5}{n^2}\cdot\frac{n(n+1)}{2}+\frac{4}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}-\frac{1}{n^4}\cdot\frac{n^2(n+1)^2}{4}\right)$$

$$=2-\frac{5}{2}+\frac{4}{3}-\frac{1}{4}$$

$$=\boxed{}$$