4.4 #45-55 odd; 75-91 odd

4.5 #5-25 odd; 33-61 odd

HW 4.4: 45-51 odd 75-91 odd

4.5 # 7-33, 41-53, 57-63 odd

$$\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))\cdot g'(x)$$

$$F(x)=\int_{2^{\cos x}}^{\ln(\arctan x)} \sqrt{t}\,dt$$

$$F(x)=\int_{2^{\cos x}}^{a} \sqrt{t}\,dt + \int_a^{\ln(\arctan x)} \sqrt{t}\,dt$$

$$= -\int_a^{2^{\cos x}} \sqrt{t}\,dt + \int_a^{\ln(\arctan x)} \sqrt{t}\,dt$$

$$F'(x) = -\sqrt{2^{\cos x}}\cdot\underbrace{2^{\cos x}\ln 2\cdot(-\sin x)}_{[2^{\cos x}]'} + \sqrt{\ln(\arctan x)}\cdot\underbrace{\frac{1}{\arctan x}\cdot\frac{1}{1+x^2}}_{[\ln(\arctan x)]'}$$

4.5 Integration by Substitution

12. $\int x^2(x^3+5)^4\,dx = \int \frac{1}{3}u^4\,du$

Let $u = x^3+5$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$= \frac{1}{15}u^5 + C$

$= \boxed{\frac{1}{15}(x^3+5)^5 + C}$

22. $\int \frac{x^2}{(16-x^3)^2}\,dx = \int -\frac{1}{3}\cdot\frac{1}{u^2}\,du$

$u = 16-x^3$

$du = -3x^2\,dx$

$-\frac{1}{3}du = x^2\,dx$

$= \int -\frac{1}{3}u^{-2}\,du$

$= \frac{1}{3}u^{-1} + C = \frac{1}{3u} + C$

$= \boxed{\frac{1}{3(16-x^3)} + C}$

4.5

50. $\int \sqrt{\tan x}\ \sec^2 x\ dx = \int u^{1/2}\, du$

$u = \tan x$

$du = \sec^2 x\, dx$

$= \frac{2}{3} u^{3/2} + C$

$= \boxed{\frac{2}{3}(\tan x)^{3/2} + C}$

54. $\int \csc^2\left(\frac{x}{2}\right) dx$

52. $\int \frac{\sin x}{\cos^3 x}\, dx = \int \frac{-du}{u^3} = \int -u^{-3}\, du$

$u = \cos x$

$du = -\sin x\, dx$

$-du = \sin x\, dx$

$= \frac{1}{2} u^{-2} + c$

$= \frac{1}{2u^2} + c$

$= \boxed{\frac{1}{2\cos^2 x} + c}$

$= \frac{1}{2} \sec^2 x + c$

58. $\int x\sqrt{2x+1}\,dx = \int \left(\frac{1}{2}u-\frac{1}{2}\right)\sqrt{u}\cdot\frac{1}{2}\,du$

$u = 2x+1 \longrightarrow u-1=2x$

$\frac{1}{2}u-\frac{1}{2}=x$

$du = 2dx$

$\frac{1}{2}du = dx$

$= \frac{1}{4}\int (u^{2/2}-1)\cdot u^{1/2}\,du$

$= \frac{1}{4}\int \left(u^{3/2}-u^{1/2}\right)du$

$= \frac{1}{4}\left(\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2}\right)+c$

$= \boxed{\frac{1}{10}(2x+1)^{5/2}-\frac{1}{6}(2x+1)^{3/2}+c}$

62. $\int \frac{2x+1}{\sqrt{x+4}}\,dx = \int \frac{2u-7}{\sqrt{u}}\,du = \int \left(\frac{2u}{u^{1/2}}-\frac{7}{u^{1/2}}\right)du$

$u = x+4 \longrightarrow u-4=x$

$du = dx$

$2u-8=2x$

$2u-7=2x+1$

$= \int \left(2u^{1/2}-7u^{-1/2}\right)du$

$= \frac{4}{3}u^{3/2}-14u^{1/2}+C$

$= \boxed{\frac{4}{3}(x+4)^{3/2}-14(x+4)^{1/2}+c}$