4.4 #45-55 odd; 75-91 odd

4.5 #5-25 odd; 33-61 odd

4.4 #45-51 odd; 75-91 odd
4.5 #7-33 odd; 41-53 odd; 57-75 odd
5.2 #1-35 odd; 43-53 odd; 61, 63
5.4 #87-107 odd
5.5 #61-68 all
5.9 #1-41 odd
Take-home quiz
Ch 5 Review pp.405-407 #17-24, 49-56,71-72, 99-106

old textbook problem #'s;
will be updated to reflect new text

Definite Integrals

66. $\int_{-2}^{4} x^2(x^3+8)^2\,dx = \int_{x=-2}^{x=4} \frac{1}{3}u^2\,du$

Let $u = x^3+8$
$du = 3x^2\,dx$
$\frac{1}{3}du = x^2\,dx$

$x=-2 \rightarrow u=(-2)^3+8=0$
$x=4 \rightarrow u=(4)^3+8=72$

$= \frac{1}{9}u^3 \Big|_{x=-2}^{x=4} = \frac{1}{9}(x^3+8)^3\Big|_{x=-2}^{x=4}$

$= \frac{1}{9}(4^3+8)^3 - \frac{1}{9}((-2)^3+8)^3$

$= \boxed{41{,}472}$

$\int_0^{72} \frac{1}{3}u^2\,du = \frac{1}{9}u^3\Big|_0^{72} = 41{,}472$

$$\int \frac{du}{u} = \ln|u| + K$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + K$$

$$\int \tan u\,du = -\ln|\cos u| + K$$

$$\int \cot u\,du = \ln|\sin u| + K$$

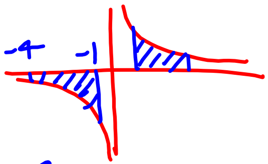
$$\int \sec u\,du = \ln|\sec u + \tan u| + K$$

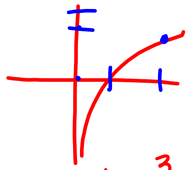
$$\int \csc u\,du = \ln|\csc u - \cot u| + K$$

$\int \frac{1}{x}dx = \ln|x| + c$

Recall:

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$





$$\int_1^3 \frac{1}{x}dx = \ln 3 - \ln 1 = \ln\frac{3}{1}$$

$$\int_{-4}^{-1} \frac{1}{x}dx = \ln|-1| - \ln|-4| = \ln 1 - \ln 4 = \boxed{-\ln 4}$$

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}dx = \int -\frac{du}{u} = -\ln|u| + c$$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$$= -\ln|\cos x| + c$$

5.2

11. $\displaystyle\int \frac{x^2+2x+3}{x^3+3x^2+9x}\,dx = \int \frac{1}{3}\cdot\frac{du}{u} = \frac{1}{3}\ln|u| + c$

$= \boxed{\frac{1}{3}\ln|x^3+3x^2+9x| + c}$

$u = x^3+3x^2+9x$

$du = (3x^2+6x+9)dx$

$\frac{1}{3}du = (x^2+2x+3)dx$

7. $\displaystyle\int \frac{x}{x^2+1}\,dx = \int \frac{1}{2}\frac{du}{u} = \frac{1}{2}\ln|x^2+1| + c$

$= \boxed{\frac{1}{2}\ln(x^2+1) + c}$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

9. $\int \frac{x^2-4}{x}\,dx = \int x\,dx - \int \frac{4}{x}\,dx$

$= \boxed{\frac{1}{2}x^2 - 4\ln|x| + C}$

$\cancel{x^{-1} \to \frac{x^{-1+1}}{-1+1}}$

boom!

5.2

34. $\int \frac{\csc^2 t}{\cot t}\,dt = \int -\frac{du}{u} = \boxed{-\ln|\cot t| + C}$

$u = \cot t$

$du = -\csc^2 t\,dt$

64. $F(x) = \int_1^{x^2} \frac{1}{t}\,dt$

Find $F'(x)$. $= \frac{1}{x^2} \cdot 2x = \frac{2}{x}$

$F(x) = \ln|t|\Big|_1^{x^2} = \ln x^2 - \ln 1 = \ln x^2$

$F'(x) = [\ln x^2]' = \frac{1}{x^2} \cdot 2x$

5.4

$\int e^x dx = e^x + C$ $\quad [e^x]' = e^x$

5.5

$[a^x]' = a^x \cdot \ln a$

$\int a^x dx = \frac{1}{\ln a} \cdot a^x + C$

5.4

94. $\int \frac{e^{1/x^2}}{x^3}\,dx = \int -\frac{1}{2}e^u\,du = -\frac{1}{2}e^u + c$

$= \boxed{-\frac{1}{2}e^{1/x^2} + c}$

$u = \frac{1}{x^2} = x^{-2}$

$du = -2x^{-3}dx = -2\cdot\frac{dx}{x^3}$

$-\frac{1}{2}du = \frac{dx}{x^3}$

104. $\int \frac{e^{2x} + 2e^x + 1}{e^x}\,dx$

$\boxed{\frac{x^m}{x^n} = x^{m-n} \quad \frac{e^{2x}}{e^x} = e^{2x-x}}$

$= \int \frac{e^{2x}}{e^x}dx + \int \frac{2e^x}{e^x}dx + \int \frac{1}{e^x}dx$

$= \int e^x dx + \int 2\,dx + \int e^{-x}dx$

$u = -x$

$du = -dx$

$\int -e^u du$

$= -e^u$

$= \boxed{e^x + 2x - e^{-x} + c}$

Integral Calculus | Test #1 | Name: KEY
Brewer | | 30 November 2016

Part I - Print the letter of the correct answer choice in the blank provided.
YOU MUST SHOW ALL OF YOUR WORK IN ORDER TO EARN FULL CREDIT.

1. d

Use differentials to approximate the value of $\sqrt[3]{7.5}$. Round your answer to four decimal places.

a. 1.9683
b. 1.9483
c. 1.9383
$\boxed{\text{d. } 1.9583}$
e. 1.9783

$f(x) = x^{1/3}$

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3(\sqrt[3]{x})^2}$

$f(c+\Delta x) \approx f(c) + f'(c)\Delta x$

$\sqrt[3]{8+(-0.5)} \approx \sqrt[3]{8} + \frac{1}{3(\sqrt[3]{8})^2} \cdot \left(-\frac{1}{2}\right)$

$= 2 + \frac{1}{3 \cdot 2^2} \cdot \frac{-1}{2} = 2 + \frac{-1}{24}$

$= \frac{48}{24} - \frac{1}{24} = \frac{47}{24} \approx 1.958\overline{3}$

2. e

Find the indefinite integral and check the result by differentiation.

$\int \frac{7z^2 + 4z - 18}{z^4}\,dz = \int (7z^{-2} + 4z^{-3} - 18z^{-4})\,dz$

$= -7z^{-1} - 2z^{-2} + 6z^{-3} + C$

$= -\frac{7}{z} - \frac{2}{z^2} + \frac{6}{z^3} + C$

a. $-\frac{7}{z} + \frac{2}{z^2} + \frac{6}{z^3}$
b. $-\frac{7}{z} - \frac{2}{z^2} + \frac{6}{z^3}$ -2
c. $-\frac{7}{z} + \frac{4}{z^2} + \frac{18}{z^3} + C$
d. $\frac{7}{z} - \frac{2}{z^2} + \frac{6}{z^3} + C$
$\boxed{\text{e. } -\frac{7}{z} - \frac{2}{z^2} + \frac{6}{z^3} + C}$

3. b

Find the indefinite integral $\int 5\sin x + 7\cos x \; dx$.

a. $-5\cos x - 7\sin x + C$ ← -5
$\boxed{\text{b. } -5\cos x + 7\sin x + C}$
c. $-7\cos x + 5\sin x + C$
d. $-5\cos^2 x + 7\sin^2 x + C$
e. $5\cos^2 x - 7\sin^2 x + C$

/30

4. C

$S_0 = 5$ $V_0 = 45$

A ball is thrown vertically upwards from a height of 5 ft with an initial velocity of 45 ft per second. How high will the ball go? Note that the acceleration of the ball is given by $a(t) = -32$ feet per second per second.

$s(t) = ?$ when $v(t) = 0$

$a(t) = -32$

$v(t) = -32t + 45$

$s(t) = -16t^2 + 45t + 5$

a. 99.9219 ft
b. 89.9219 ft
c. 36.6406 ft
d. 28.7305 ft
e. 102.9219 ft

$-32t + 45 = 0$

$t = \frac{45}{32}$

≈ 1.40625

$s\left(\frac{45}{32}\right) = -16\left(\frac{45}{32}\right)^2 + 45\left(\frac{45}{32}\right) + 5$

$= 36.640625$

5. d

Use the summation formulas to rewrite the expression $\sum_{k=1}^{n} \frac{24k(k-1)}{n^9}$ without the summation notation.

a. $\frac{4}{n^6} - \frac{4}{n^8}$

b. $\frac{4}{n^7} - \frac{1}{n^8}$

c. $\frac{8}{n^7} - \frac{4}{n^8}$

d. $\frac{8}{n^6} - \frac{8}{n^8}$

e. $\frac{8}{n^6} + \frac{2}{n^9}$

$\frac{24}{n^9} \sum_{k=1}^{n} (k^2 - k) = \frac{24}{n^9} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{24}{n^9} \cdot \frac{n(n+1)}{2}$

$= \frac{4}{n^8}(2n^2 + 3n + 1) - \frac{12}{n^8}(n+1)$

$= \frac{8n^2 + 12n + 4 - 12n - 12}{n^8} = \frac{8n^2 - 8}{n^8} = \frac{8}{n^6} - \frac{8}{n^8}$

/20

6. e

Use the summation formulas to rewrite the expression $\sum_{i=1}^{n} \frac{6i^3 - 4i}{n^4}$ without the summation notation.

$\frac{6}{n^4} \cdot \frac{n^2(n+1)^2}{4} - \frac{4}{n^4} \cdot \frac{n(n+1)}{2} \cdot \frac{2}{2}$

$\frac{n(n+1)}{4n^4}\left[6n(n+1) - 8\right]$

a. $\dfrac{(n+1)\left[6n(n+1)-2\right]}{4n^3}$

b. $\dfrac{(n+1)\left[6(n+1)-4\right]}{4n^3}$

c. $\dfrac{(n+1)\left[6n(n+1)-4\right]}{4n^2}$

d. $\dfrac{(n+1)\left[n(n+1)+4\right]}{4n^3}$

e. $\boxed{\dfrac{(n+1)\left[6n(n+1)-8\right]}{4n^3}}$

7. D

$\int_0^1 (3x-2)^2\, dx =$ $\int_0^1 (9x^2 - 12x + 4)\,dx = 3x^3 - 6x^2 + 4x \Big|_0^1 = 3 - 6 + 4$

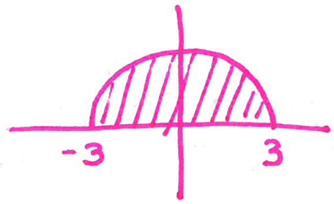
(A) $-\frac{7}{3}$ (B) $-\frac{7}{9}$ (C) $\frac{1}{9}$ $\boxed{\text{(D) } 1}$ (E) 3

/20

8. e

Sketch the region whose area is given by the definite integral and then use a geometric formula to evaluate the integral.

$$\int_{-3}^{3} \sqrt{9-t^2}\, dt$$



$\frac{1}{2}\pi(3)^2$

a. 9π

b. $\frac{9}{8}\pi$

c. $\frac{9}{2}$

d. $\frac{9}{4}\pi$

e. $\frac{9}{2}\pi$

9. c

Write the limit $\lim\limits_{|\Delta x| \to 0} \sum\limits_{i=1}^{n} 5c_i\left(6-c_i\right)^2 \Delta x_i$, as a definite integral on the interval $[0,4]$ where c_i is any point in the i^{th} subinterval.

a. $\int 5x(6-x)^4 dx$

b. $\int 5c_i\left(6-c_i\right)^2 \Delta x_i$

c. $\int_0^4 5x(6-x)^2 dx$

d. $\int_0^{c_i} 5c_i\left(6-c_i\right)^2 \Delta x_i$

e. $\int_4^0 5x^2(6-x)^4 dx$

/20

10. E

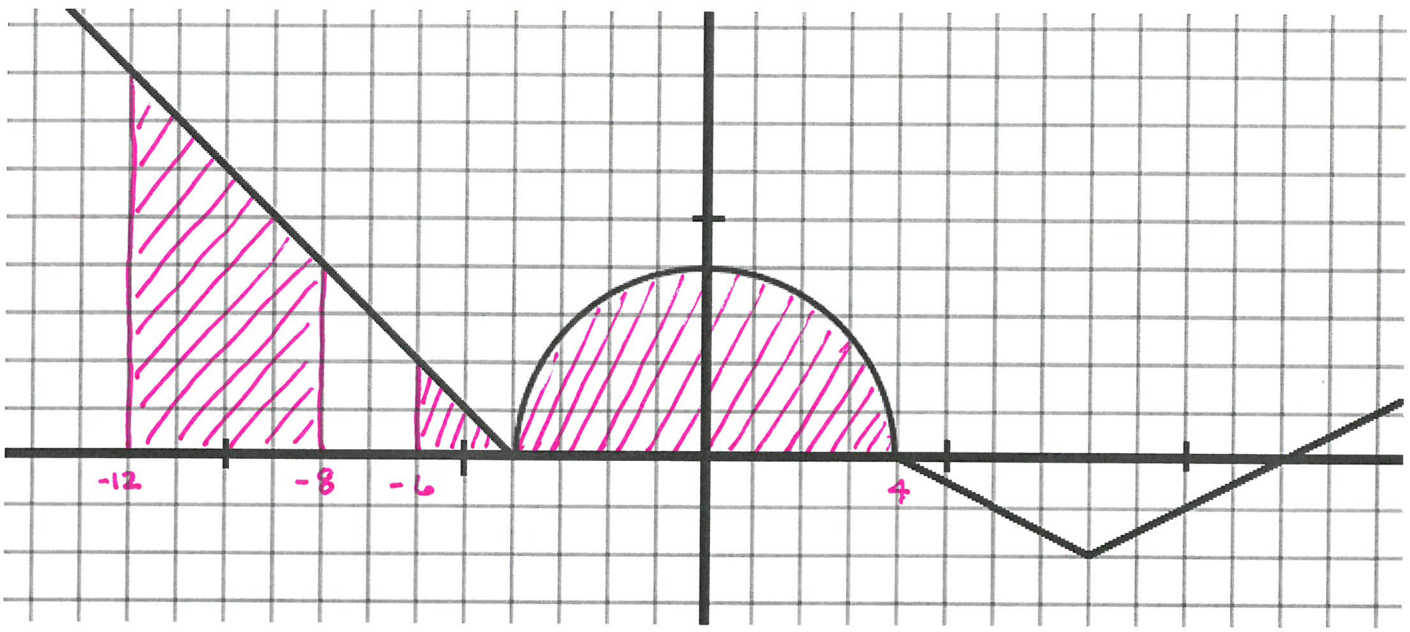
If $\int_1^{10} f(x)\,dx = 4$ and $\int_{10}^{3} f(x)\,dx = 7$, then $\int_1^{3} f(x)\,dx =$

(A) −3 (B) 0 (C) 3 (D) 10 $\boxed{\text{(E) } 11}$

$\int_1^{10} = \int_1^{3} + \int_3^{10} \Rightarrow \int_1^{3} = \int_1^{10} - \int_3^{10} = \int_1^{10} + \int_{10}^{3} = 4 + 7$

/10

Part II - Use the graph to determine the definite integrals. Assume the scale for the graph is 1 to 1.



11. $\int_{-12}^{-8} f(x)dx$ $= \frac{1}{2}(4+8)4$

$= \boxed{24}$

/5

12. $\int_{-6}^{4} f(x)dx$ $= \frac{1}{2}(2)(2) + \frac{1}{2}\pi(4)^2$

$= \boxed{2 + 8\pi} \approx 27.13274$

/5

/20