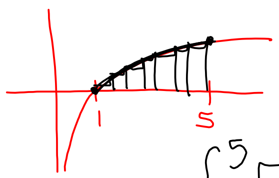
- 7.1 #1-9 odd; 19, 37 — area between curves
- 7.2 #11, 13, 17, 19, 21, 25, 29, 37 — volume of solids of revolution
- 7.4 #7,9,19, 37,39 — arc length & surface area of solids of revolution
- 8.1 #5-45 odd — basic integration techniques
- 8.2 #1-29 odd — integration by parts

18. $y = \ln x$, $[1,5]$

$s = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$



$y = \ln x$

$y' = \frac{1}{x}$

$$s = \int_1^5 \sqrt{1+\left(\frac{1}{x}\right)^2}\,dx$$

$$s = \int_1^5 \sqrt{\left(\frac{1}{x^2}\right)\left(x^2+1\right)}\,dx$$

$$= \int_1^5 \frac{1}{x}\sqrt{x^2+1}\,dx = \int_1^5 \frac{\sqrt{x^2+1}}{x}\cdot\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}\,dx$$

$$= \int_1^5 \frac{x^2+1}{x\sqrt{x^2+1}}\,dx = \int_1^5 \frac{x^{\cancel{2}}\,dx}{\cancel{x}\sqrt{x^2+1}} + \int_1^5 \frac{dx}{x\sqrt{x^2+1}}$$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

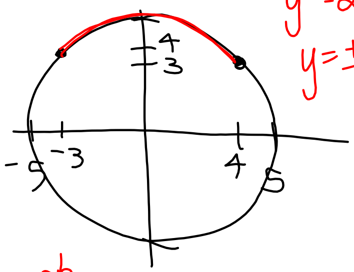
$\int_{x=1}^{5} \frac{1}{2}u^{-1/2}\,du$

$x = \tan\theta$

$\tan^2\theta + 1$

$\sec^2\theta$

32. Find arc length from $(-3,4)$ clockwise to $(4,3)$ along the circle $x^2+y^2=25$.



$y^2=25-x^2 \qquad 2x+2yy'=0$

$y=\pm\sqrt{25-x^2} \qquad y'=\frac{-x}{y}$

$y=(25-x^2)^{1/2}$

$y'=\frac{1}{2}(25-x^2)^{-1/2}\cdot(-2x)$

$y=\frac{-x}{\sqrt{25-x^2}}$

$S=\int_a^b\sqrt{1+\boxed{f'(x)}^2}\,dx$

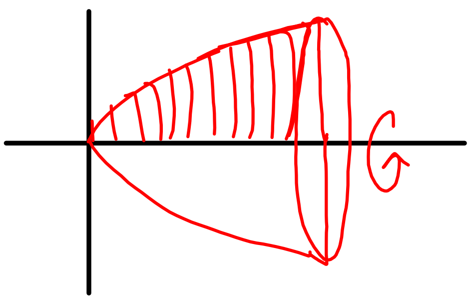
$S=\int_{-3}^{4}\sqrt{1+\left(\frac{-x}{\sqrt{25-x^2}}\right)^2}\,dx$

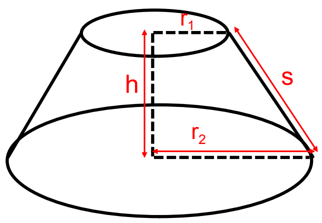
$=\int_{-3}^{4}\sqrt{1+\frac{x^2}{25-x^2}}\,dx=\int_{-3}^{4}\sqrt{\frac{25-x^2}{25-x^2}+\frac{x^2}{25-x^2}}\,dx$

$=\int_{-3}^{4}\sqrt{\frac{25}{25-x^2}}\,dx=\int_{-3}^{4}\frac{5\,dx}{\sqrt{25-x^2}}=5\arcsin\frac{x}{5}\Big|_{-3}^{4}$

$=\boxed{5\arcsin\frac{4}{5}-5\arcsin\frac{3}{5}}$

Area of a Surface of Revolution





Truncated Cone:

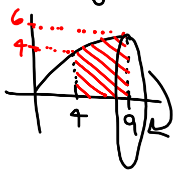
$A=2\pi\cdot r_{avg}\cdot s$

lateral area

$$S=2\pi\int_a^b r(x)\sqrt{1+[f'(x)]^2}\,dx$$

$$S = 2\pi\int_a^b r(x)\sqrt{1+[f'(x)]^2}\,dx$$

34. $y = 2\sqrt{x}$, $[4,9]$ $r(x) = 2\sqrt{x}$; $f'(x) = \frac{1}{\sqrt{x}}$
revolve about x-axis



$$\int_4^9 2\pi(2\sqrt{x})\sqrt{1+\left(\frac{1}{\sqrt{x}}\right)^2}\,dx$$

$$= \int_4^9 4\pi\sqrt{x}\sqrt{1+\frac{1}{x}}\,dx$$

$$= \int_4^9 4\pi\sqrt{x+1}\,dx \qquad \begin{array}{l} u = x+1 \\ du = dx \end{array}$$

$$= \int_{x=4}^{9} 4\pi u^{1/2}\,du = 4\pi\cdot\frac{2}{3}(x+1)^{3/2}\Big|_4^9$$

$$\frac{8\pi}{3}\sqrt{10^3} - \frac{8\pi}{3}\sqrt{5^3} = \boxed{\frac{80\pi}{3}\sqrt{10} - \frac{40\pi}{3}\sqrt{5}}$$

8.1 Basic Integration Rules

$$\int\frac{4}{x^2+9}\,dx \qquad \int\frac{4x}{x^2+9}\,dx$$

$$= \frac{4}{3}\arctan\frac{x}{3} + C$$

$u = x^2+9$
$du = 2x\,dx$
$2du = 4x\,dx$

$$\int\frac{2du}{u} = \boxed{2\ln|x^2+9| + C}$$

$$\int \frac{4x^2}{x^2+9}\,dx \qquad \cancel{u = x^2} \quad \cancel{du = 2x\,dx}$$

$$= 4\int \frac{x^2+9-9}{x^2+9}\,dx = 4\int dx - \int \frac{36\,dx}{x^2+9}$$

$$= 4x - \frac{36}{3}\arctan\frac{x}{3} + C$$

$$= \boxed{4x - 12\arctan\frac{x}{3} + C}$$

$$\int \frac{1}{1+e^x}\,dx \qquad \boxed{\cancel{\frac{a}{b+c} = \frac{a}{b} + \frac{a}{c}}}$$

$$= \int \frac{1+e^x-e^x}{1+e^x}\,dx$$

$$= \int \frac{1+e^x}{1+e^x}\,dx - \int \frac{e^x}{1+e^x}\,dx = \int 1\,dx - \int \frac{e^x\,dx}{1+e^x}$$

$$= \boxed{x - \ln|1+e^x| + C}$$

$u = 1+e^x$

$du = e^x\,dx$

$$\int \frac{du}{u} = \ln|u|$$

$$\int \tan^2 2x \, dx$$

$$= \int (\sec^2 2x - 1)\, dx$$

$$= \boxed{\frac{1}{2}\tan 2x - x + C}$$

$$\frac{\sin^2 x + \cos^2 x = 1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\tan^2 x = \sec^2 x - 1$$