- 8.1 #5-45 odd basic integration techniques
- 8.2 #1-29 odd integration by parts

$$\int \cot x \ln(\sin x)\,dx = \int u\,du = \frac{1}{2}u^2 + c$$

$$= \int \frac{\cos x}{\sin x} \ln(\sin x)\,dx \qquad = \boxed{\frac{1}{2}\left(\ln(\sin x)\right)^2 + c}$$

$$u = \ln(\sin x)$$

$$du = \frac{1}{\sin x} \cdot \cos x\,dx$$

8.2 Integration by Parts

$$\frac{d}{dx}[uv] = u \cdot \frac{d}{dx}[v] + v \cdot \frac{d}{dx}[u] = uv' + vu'$$

$$\frac{d(uv)}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

$$\frac{dv}{dx} \cdot dx = dv$$

Integrating both sides w.r.t. x yields:

$$uv = \int uv'dx + \int vu'dx$$

$$uv = \int u\,dv + \int v\,du$$

Rearranging yields:

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$$\int xe^x dx$$

$$\int u\,dv = uv - \int v\,du$$

$u = x$ $\quad dv = e^x dx$

$du = dx$ $\quad v = e^x$

$$= xe^x - \int e^x dx = \boxed{xe^x - e^x + C}$$

6. $\int x^2 e^{2x}\,dx$

$u = x^2$ $\quad \int dv = \int e^{2x}\,dx$ $\quad u = 2x$, $du = 2dx$, $\frac{1}{2}du = dx$, $\int \frac{1}{2} e^u du$

$du = 2x\,dx$ $\quad v = \frac{1}{2}e^{2x}$

$= \frac{1}{2}x^2e^{2x} - \int x e^{2x}\,dx$

$u = x$ $\quad dv = e^{2x}dx$

$du = dx$ $\quad v = \frac{1}{2}e^{2x}$

$= \frac{1}{2}x^2e^{2x} - \left(\frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}dx\right)$

$= \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \int \frac{1}{2}e^{2x}dx$ $\quad \frac{1}{2}\int e^{2x}dx$, $\frac{1}{2}\left(\frac{1}{2}e^{2x}\right)$

$= \boxed{\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C}$

16. $\int x^4 \ln x\,dx$

$u = \ln x$ $\quad dv = x^4dx$

$du = \frac{1}{x}dx$ $\quad v = \frac{1}{5}x^5$

$= \frac{1}{5}x^5\ln x - \int \frac{1}{5}x^4dx$

$= \boxed{\frac{1}{5}x^5\ln x - \frac{1}{25}x^5 + C}$

34. $\int 4\arccos x\,dx = 4x\arccos x - \int \frac{-4x\,dx}{\sqrt{1-x^2}}$

$u = \arccos x$ $\quad dv = 4\,dx$

$du = \frac{-dx}{\sqrt{1-x^2}}$ $\quad V = 4x$

$= 4x\arccos x + \int \frac{4x\,dx}{\sqrt{1-x^2}}$

$u = 1-x^2$

$du = -2x\,dx$

$-2du = 4x\,dx$

$= 4\arccos x + \int \frac{-2du}{\sqrt{u}} = 4\arccos x - 2\int u^{-1/2}\,du$

$= 4\arccos x - 2\left(2u^{1/2}\right) + C$

$= \boxed{4\arccos x - 4\sqrt{1-x^2} + C}$

30. $\int x^2\cos x\,dx = x^2\sin x - \int 2x\sin x\,dx$

$u = 2x$ $\quad dv = \sin x\,dx$

$du = 2dx$ $\quad V = -\cos x$

$u = x^2$ $\quad dv = \cos x\,dx$

$du = 2x\,dx$ $\quad v = \sin x$

$= x^2\sin x - \left(-2x\cos x - \int -2\cos x\,dx\right)$

$= x^2\sin x + 2x\cos x - \int 2\cos x\,dx$

$= \boxed{x^2\sin x + 2x\cos x - 2\sin x + C}$