- 8.1 #5-45 odd — basic integration techniques
- 8.2 #1-29 odd — integration by parts

- 8.3 #1-11odd; 19-31odd; 47-63odd — trigonometric integrals
- 8.4 #5-15odd; 21-39odd — trigonometric substitution

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$\int u\,dv = uv - \int v\,du$

12. $\int \sin^2 2x \, dx$

$= \int \left(\frac{1}{2} - \frac{1}{2}\cos 4x\right) dx$

$= \boxed{\frac{1}{2}x - \frac{1}{8}\sin 4x + C}$

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 (x) = \frac{1}{2} - \frac{1}{2}\cos 2(x)$

$\sin^2(2x) = \frac{1}{2} - \frac{1}{2}\cos 4x$

26. $\int \tan^2 x \, dx = \int (\sec^2 x - 1)\, dx$

$= \boxed{\tan x - x + C}$

38. $\int \frac{\tan^2 x}{\sec^5 x} dx = \int \frac{\sec^2 x - 1}{\sec^5 x} dx$

$= \int \left(\frac{1}{\sec^3 x} - \frac{1}{\sec^5 x} \right) dx$

$= \int (\cos^3 x - \cos^5 x) dx = \int (\cos^2 x - \cos^4 x) \cos x \, dx$

38. $\displaystyle\int \frac{\tan^2 x}{\sec^5 x}\,dx = \int \frac{\sin^2 x}{\cos^2 x}\cdot\frac{\cos^5 x}{1}\,dx =$

$= \int \sin^2 x \cos^3 x\,dx =$

$= \int \sin^2 x \underbrace{\cos^2 x}\cos x\,dx =$

$= \int \sin^2 x(1-\sin^2 x)\cos x\,dx =$

$= \int (\sin^2 x - \sin^4 x)\cos x\,dx = \int (u^2 - u^4)\,du$

$u = \sin x$
$du = \cos x\,dx$

$= \frac{1}{3}u^3 - \frac{1}{5}u^5 + C$

$= \boxed{\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C}$

16. $\int x^2 \sin^2 x\,dx$

$u = \sin^2 x \qquad dv = x^2\,dx$
$du = 2\sin x\cos x\,dx \qquad v = \frac{1}{3}x^3$

$u = x^2 \qquad dv = \sin^2 x\,dx$
$du = 2x\,dx \qquad = \frac{1}{2} - \frac{1}{2}\cos 2x$
$v = \frac{1}{2}x - \frac{1}{4}\sin 2x$

$= x^2\left(\frac{1}{2}x - \frac{1}{4}\sin 2x\right) - \int 2x\left(\frac{1}{2}x - \frac{1}{4}\sin 2x\right)dx$

$= \frac{1}{2}x^3 - \frac{x^2}{4}\sin 2x - \int x^2\,dx + \frac{1}{2}\int x\sin 2x$

$u = x \qquad dv = \sin 2x$
$du = dx \qquad v = -\frac{1}{2}\cos 2x$

$= \frac{1}{2}x^3 - \frac{1}{4}x^2\sin 2x - \frac{1}{3}x^3 + \frac{1}{2}\left(-\frac{1}{2}x\cos 2x - \int -\frac{1}{2}\cos 2x\,dx\right)$

$= \frac{1}{6}x^3 - \frac{1}{4}x^2\sin 2x - \frac{1}{4}x\cos 2x + \frac{1}{4}\int \cos 2x\,dx$

$= \boxed{\frac{1}{6}x^3 - \frac{1}{4}x^2\sin 2x - \frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x + C}$