

- 8.1 #5-45 odd
- 8.2 #1-29 odd

basic integration techniques
integration by parts

- 8.3 #1-11 odd; 19-31 odd; 47-63 odd
- 8.4 #5-15 odd; 21-39 odd

trigonometric integrals
trigonometric substitution

$$\begin{aligned}\sin^2 x + \cos^2 x &= 1 \\ \tan^2 x + 1 &= \sec^2 x \\ \cot^2 x + 1 &= \csc^2 x\end{aligned}$$

$$\begin{aligned}\sin 2x &= 2 \sin x \cos x \\ \cos 2x &= 2 \cos^2 x - 1 \\ &= 1 - 2 \sin^2 x\end{aligned}$$

$$\int u dv = uv - \int v du$$

$$\begin{aligned}12. \int \sin^2 2x \, dx \\ &= \int \left(\frac{1}{2} - \frac{1}{2} \cos 4x \right) dx \\ &= \frac{1}{2}x - \frac{1}{8} \sin 4x + C\end{aligned}$$

$$\begin{aligned}\cos 2x &= 1 - 2 \sin^2 x \\ 2 \sin^2 x &= 1 - \cos 2x \\ \sin^2 x &= \frac{1}{2} - \frac{1}{2} \cos 2x \\ \sin^2(2x) &= \frac{1}{2} - \frac{1}{2} \cos 4x\end{aligned}$$

$$26. \int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx$$
$$= \boxed{\tan x - x + C}$$

$$38. \int \frac{\tan^2 x}{\sec^5 x} \, dx = \int \frac{\sec^2 x - 1}{\sec^5 x} \, dx$$
$$= \int \left(\frac{1}{\sec^3 x} - \frac{1}{\sec^5 x} \right) \, dx$$
$$= \int (\cos^3 x - \cos^5 x) \, dx = \int (\cos^2 x - \cos^4 x) \cos x \, dx$$

$$\begin{aligned}
 38. \int \frac{\tan^2 x}{\sec^5 x} dx &= \int \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^5 x}{1} dx = \\
 &= \int \sin^2 x \cos^3 x dx = \\
 &= \int \sin^2 x \underbrace{\cos^2 x}_{1-\sin^2 x} \cos x dx = \\
 &= \int \sin^2 x (1-\sin^2 x) \cos x dx = \\
 &= \int (\sin^2 x - \sin^4 x) \cos x dx = \int (u^2 - u^4) du \\
 &\quad \begin{array}{l} u = \sin x \\ du = \cos x dx \end{array} \\
 &= \frac{1}{3} u^3 - \frac{1}{5} u^5 + C \\
 &= \boxed{\frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C}
 \end{aligned}$$

$$16. \int x^2 \sin^2 x dx$$

$$\begin{array}{ll}
 u = \sin^2 x & dv = x^2 dx \\
 du = 2 \sin x \cos x dx & v = \frac{1}{3} x^3 \\
 \hline
 u = x^2 & dv = \sin^2 x dx \\
 du = 2x dx & = \frac{1}{2} - \frac{1}{2} \cos 2x \\
 & v = \frac{1}{2} x - \frac{1}{4} \sin 2x
 \end{array}$$

$$\begin{aligned}
 &= x^2 \left(\frac{1}{2} x - \frac{1}{4} \sin 2x \right) - \int 2x \left(\frac{1}{2} x - \frac{1}{4} \sin 2x \right) dx \\
 &= \frac{1}{2} x^3 - \frac{x^2}{4} \sin 2x - \int x^2 dx + \frac{1}{2} \int x \sin 2x
 \end{aligned}$$

$$\begin{array}{ll}
 u = x & dv = \sin 2x \\
 du = dx & v = -\frac{1}{2} \cos 2x
 \end{array}$$

$$\begin{aligned}
 &= \frac{1}{2} x^3 - \frac{1}{4} x^2 \sin 2x - \frac{1}{3} x^3 + \frac{1}{2} \left(-\frac{1}{2} x \cos 2x - \int -\frac{1}{2} \cos 2x dx \right) \\
 &= \frac{1}{6} x^3 - \frac{1}{4} x^2 \sin 2x - \frac{1}{4} x \cos 2x + \frac{1}{4} \int \cos 2x dx \\
 &= \boxed{\frac{1}{6} x^3 - \frac{1}{4} x^2 \sin 2x - \frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + C}
 \end{aligned}$$