- 8.1 #5-45 odd — basic integration techniques
- 8.2 #1-29 odd — integration by parts

- 8.3 #1-11odd; 19-31odd; 47-63odd — trigonometric integrals
- 8.4 #5-15odd; 21-39odd — trigonometric substitution

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$\int u\,dv = uv - \int v\,du$

$\int \sec 4x\,dx = x\sec 4x - \int 4x\sec 4x\tan 4x\,dx$

$\frac{1}{\cos 4x} \cdot \frac{\sin 4x}{\cos 4x}$

$u = \sec 4x$ $\quad dv = dx$

$du = 4\sec 4x\tan 4x$ $\quad v = x$

$= x\sec 4x - \int 4x\,\frac{\sin 4x}{\cos^2 4x}\,dx$

$\cancel{u = 4x}$

$\cancel{du = 4dx}$

$\cancel{\int dv = \int \frac{\sin 4x}{\cos^2 4x}\,dx}$

$\cancel{v = \frac{1}{4\cos 4x}}$

$p = \cos 4x$

$dp = -4\sin 4x\,dx$

$-\frac{1}{4}dp = \sin 4x\,dx$

$-\frac{1}{4}\int \frac{dp}{p^2}$

$= \frac{1}{4}p^{-1} = \frac{1}{4\cos 4x}$

$$\int \sec 4x\,dx = \int \frac{dx}{\cos 4x} = \int \frac{dx}{2\cos^2(2x)-1}$$

$$= \int \frac{dx}{2(\cos 2x)^2 - 1} = \int \frac{dx}{2(2\cos^2 x - 1)^2 - 1}$$

$$2(4\cos^4 x - 4\cos^2 x + 1) - 1$$

$$= \int \frac{dx}{8\cos^4 x - 8\cos^2 x + 1} = \frac{1}{8}\int \frac{dx}{\cos^4 x - \cos^2 x + \frac{1}{8}}$$

$$\int \sec 4x\,dx = \int \frac{(1)\,dx}{\cos 4x}$$

$$= \int \frac{(\sin 4x - \sin 4x + 1)}{\cos 4x}\,dx$$

$$= \int \frac{\sin 4x + 1}{\cos 4x}\,dx - \int \frac{\sin 4x}{\cos 4x}\,dx$$

$$-\frac{du}{u}$$

$u = \sin 4x + 1$

$du = \cos x$

$$\int \sec 4x \, dx \cdot \frac{\sec 4x + \tan 4x}{\sec 4x + \tan 4x}$$

$$= \int \frac{\sec^2 4x + \sec 4x \tan 4x}{\sec 4x + \tan 4x} \, dx$$

$u = \sec 4x + \tan 4x$

$du = 4 \sec 4x \tan 4x + 4 \sec^2 4x$

$$\int \frac{1}{4} \frac{du}{u} = \frac{1}{4} \ln \left| \sec 4x + \tan 4x \right| + C$$

8.4 Trig Substitution

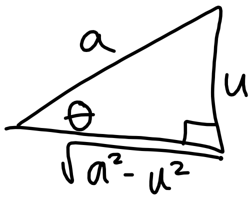
$\sqrt{a^2 - u^2} = \sqrt{a^2 - a^2 \sin^2\theta} = a\sqrt{1 - \sin^2\theta}$

$= a\sqrt{\cos^2\theta}$

$= a\cos\theta$

$u = a\sin\theta$

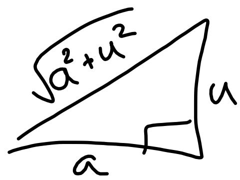
$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$



$\sqrt{a^2 + u^2} =$

$u = a\tan\theta$

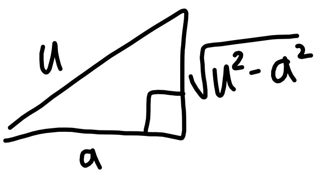
$, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$



$\sqrt{u^2 - a^2} = \begin{cases} +a\tan\theta, & u > a \\ -a\tan\theta, & u < -a \end{cases}$

$u = a\sec\theta$

$0 \le \theta < \frac{\pi}{2}$ or $\frac{\pi}{2} < \theta \le \pi$



6. $\displaystyle\int \frac{10}{x^2\sqrt{25-x^2}}\,dx = \int \frac{10\cdot 5\cos\theta\, d\theta}{(5\sin\theta)^2\sqrt{25-(5\sin\theta)^2}}$

$25-25\sin^2\theta$

$25(1-\sin^2\theta)$

$\sqrt{25\cos^2\theta}$

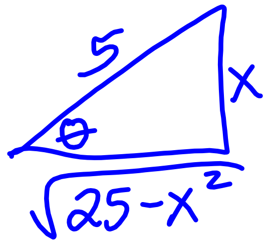
$x = 5\sin\theta \longrightarrow \sin\theta = \frac{x}{5}$

$dx = 5\cos\theta\, d\theta$

$$\int \frac{\overset{2}{\cancel{50}}\cos\theta\, d\theta}{\cancel{25}\sin^2\theta\cdot 5\cos\theta} = \frac{2}{5}\int \frac{d\theta}{\sin^2\theta} = \frac{2}{5}\int \csc^2\theta\, d\theta$$

$$= -\frac{2}{5}\cot\theta + C$$

$$= \boxed{-\frac{2}{5}\cdot\frac{\sqrt{25-x^2}}{x} + C}$$



12. $\displaystyle\int \frac{x^3\,dx}{\sqrt{x^2-4}}$

Let $x = 2\sec\theta \Rightarrow dx = 2\sec\theta\tan\theta\, d\theta$

$x^2 = 4\sec^2\theta$

$\sqrt{x^2-4} = \sqrt{4(\sec^2\theta - 1)}$

$= \sqrt{4\tan^2\theta}$

$= 2\tan\theta$

$$\int \frac{(2\sec\theta)^3\cdot \cancel{2}\sec\theta\,\cancel{\tan\theta}\, d\theta}{\cancel{2}\,\cancel{\tan\theta}} = \int 8\sec^4\theta\, d\theta$$

$$= \int 8\,\underbrace{\sec^2\theta}\,\sec^2\theta\, d\theta$$

$u = \tan\theta$

$du = \sec^2\theta$

$$= \int 8(\tan^2\theta + 1)\sec^2\theta\, d\theta$$

$$= 8\int (u^2+1)\,du = 8\left(\frac{1}{3}u^3 + u\right) + C$$