- 8.1 #5-45 odd — basic integration techniques
- 8.2 #1-29 odd — integration by parts $\int u\,dv = uv - \int v\,du$

- 8.3 #1-11odd; 19-31odd; 47-63odd — trigonometric integrals
- 8.4 #5-15odd; 21-39odd — trigonometric substitution
- 8.5 #11-21 odd — partial fractions

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

Important things coming up:

Test 3, Part B Retest (Volume of solids of revolution) - Fri. Feb 3

Test 3, Part A Retest (Area & arc length) - see me to schedule

Take-home Quiz due Wed. Feb 8

Test 4 (integration techniques) - Thurs. Feb 9

Final Exam - Thurs. Feb 16

$0 \le K < \frac{\pi}{2}$, $y = \cos x$ area from K to $\frac{\pi}{2}$ is 0.1

$$\int_K^{\pi/2} \cos x\,dx = 0.1$$

$$\sin\frac{\pi}{2} - \sin K = 0.1$$

$$1 - 0.1 = \sin K$$

$$0.9 = \sin K$$

$$\arcsin(0.9) = K \approx \boxed{1.120}$$

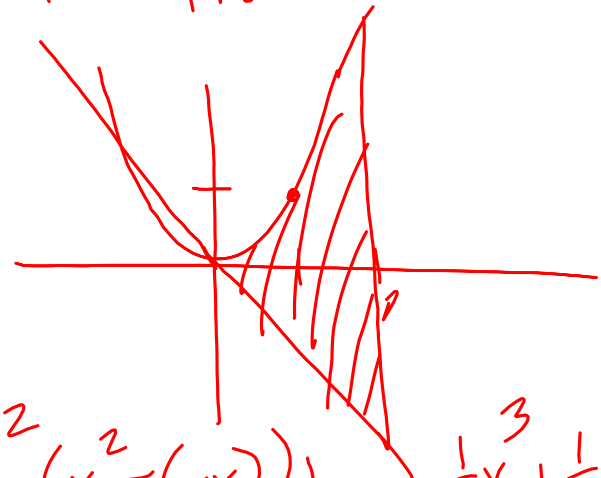
$y = x^2$ & $y = -x$ from 0 to 2 ?

$x^2 = -x$

$x^2 + x = 0$

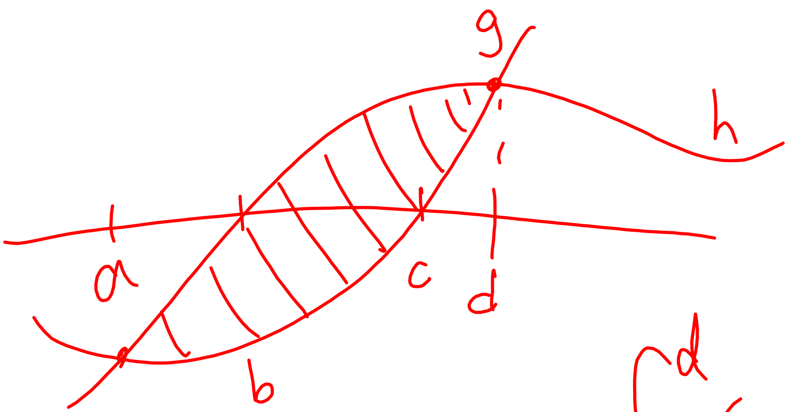
$x(x+1) = 0$

$x = 0, -1$



$$\int_0^2 \left(x^2 - (-x)\right) dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 \Big|_0^2$$

$$= \frac{8}{3} + \frac{2(3)}{3} = \boxed{\frac{14}{3}}$$



$$\int_a^d \left(h(x) - g(x)\right) dx$$

$y = e^{-x^2}$ $[0,2]$ $\int_a^b \sqrt{1 + (f'(x))^2}$

$y' = e^{-x^2} \cdot (-2x)$

$(y')^2 = \left(-2xe^{-x^2}\right)^2 = 4x^2 e^{-2x^2}$

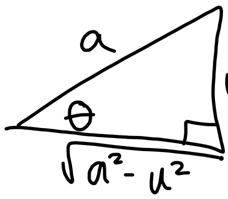
$(a^m)^n = a^{mn}$

8.4 Trig Substitution

$\sqrt{a^2 - u^2} =$

$u = a\sin\theta$

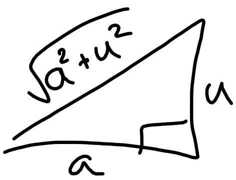
$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$



$\sqrt{a^2 + u^2} =$

$u = a\tan\theta$

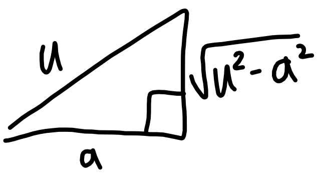
$, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$



$\sqrt{u^2 - a^2} = \begin{cases} +a\tan\theta, & u > a \\ -a\tan\theta, & u < -a \end{cases}$

$u = a\sec\theta$

$0 \le \theta < \frac{\pi}{2}$ or $\frac{\pi}{2} < \theta \le \pi$



16. $\int \frac{x^2 dx}{(1+x^2)^2}$; $x = \tan\theta$; $x^2 = \tan^2\theta$
$dx = \sec^2\theta \, d\theta$

$$\int \frac{\tan^2\theta \sec^2\theta \, d\theta}{(1+\tan^2\theta)^2} = \int \frac{\tan^2\theta \sec^2\theta \, d\theta}{(\sec^2\theta)^2}$$

$$= \int \frac{\tan^2\theta \sec^2\theta \, d\theta}{\sec^4\theta} = \int \frac{\tan^2\theta}{\sec^2\theta} d\theta =$$

$$= \int \frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{\cos^2\theta}{1} d\theta = \int \sin^2\theta \, d\theta$$

$$= \int \frac{1-\cos 2\theta}{2} d\theta = \int \left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right) d\theta =$$

Let $u = 2\theta$, $du = 2d\theta$, $\frac{1}{2}du = d\theta$, $\int -\frac{1}{4}\cos u \, du$

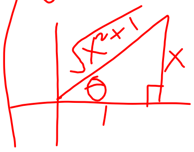
$$= \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C$$

$x = \tan\theta$
$\theta = \tan^{-1} x$

$$= \frac{1}{2}\arctan x - \frac{1}{4} \cdot 2\sin\theta\cos\theta + C$$

$$= \frac{1}{2}\arctan x - \frac{1}{2} \cdot \frac{x}{\sqrt{x^2+1}} \cdot \frac{1}{\sqrt{x^2+1}} + C$$

$$= \boxed{\frac{1}{2}\arctan x - \frac{x}{2(x^2+1)} + C}$$



30. $\int \frac{dx}{x\sqrt{4x^2+16}}$

$x = 2\tan\theta \rightarrow dx = 2\sec^2\theta \, d\theta$
$x^2 = 4\tan^2\theta$
$4x^2+16 = 16\tan^2\theta + 16$
$= 16(\tan^2\theta + 1)$
$= 16\sec^2\theta$
$\sqrt{4x^2+16} = 4\sec\theta$

$$= \int \frac{2\sec^2\theta \, d\theta}{2\tan\theta(4\sec\theta)}$$

$$= \frac{1}{4}\int \frac{\sec\theta \, d\theta}{\tan\theta}$$

$$= \frac{1}{4}\int \frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} d\theta = \frac{1}{4}\int \csc\theta \, d\theta$$

$$= \frac{1}{4}\int \frac{\csc\theta}{1} \cdot \frac{\csc\theta + \cot\theta}{\csc\theta + \cot\theta} d\theta$$

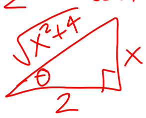
$u = \csc\theta + \cot\theta$
$du = -\csc^2\theta - \csc\theta\cot\theta \, d\theta$

$$= \frac{1}{4}\int \frac{\csc^2\theta + \csc\theta\cot\theta}{\csc\theta + \cot\theta} d\theta$$

$$= -\frac{1}{4}\int \frac{du}{u} = -\frac{1}{4}\ln|u| + C$$

$$= -\frac{1}{4}\ln|\csc\theta + \cot\theta| + C$$

$x = 2\tan\theta$
$\frac{x}{2} = \tan\theta$



$$= \boxed{-\frac{1}{4}\ln\left|\frac{\sqrt{x^2+4}}{x} + \frac{2}{x}\right| + C}$$

40. $\int x \arcsin x \, dx = \int (\sin\theta) \cdot \theta \cdot \cos\theta \, d\theta$

$x = \sin\theta$

$\arcsin x = \theta$

$dx = \cos\theta \, d\theta$

$\int \theta \cdot \frac{1}{2}\sin 2\theta \, d\theta$

$u = \theta$ $\quad dv = \frac{1}{2}\sin 2\theta \, d\theta$

$du = d\theta$ $\quad v = -\frac{1}{4}\cos 2\theta$

$-\frac{1}{4}\theta\cos 2\theta - \int -\frac{1}{4}\cos 2\theta \, d\theta$

$-\frac{1}{4}\theta\cos 2\theta + \frac{1}{8}\sin 2\theta + C$

$= -\frac{1}{4}\arcsin x \left(\cos^2\theta - \sin^2\theta\right) + \frac{1}{8} \cdot 2\sin\theta\cos\theta + C$

$= -\frac{1}{4}\arcsin x \left((1-x^2) - x^2\right) + \frac{1}{4} \cdot x\sqrt{1-x^2} + C$

$$= \boxed{-\frac{1}{4}(1-2x^2)\arcsin x + \frac{x}{4}\sqrt{1-x^2} + C}$$

40. $\int x \arcsin x \, dx$

$u = \arcsin x$ $\quad dv = x\,dx$

$du = \frac{1 \, dx}{\sqrt{1-x^2}}$ $\quad v = \frac{1}{2}x^2$