13. $\displaystyle\int_1^e \frac{(\ln x)^2}{x^2}\,dx$

$u = (\ln x)^2 \qquad dv = \frac{dx}{x^2} = x^{-2}dx$

$du = 2\ln x \cdot \frac{1}{x}dx \qquad v = -\frac{1}{x}$

$$= -\frac{(\ln x)^2}{x} + \int \frac{+2\ln x}{x^2}dx$$

$u = 2\ln x \qquad dv = \frac{dx}{x^2}$

$du = \frac{2}{x}dx \qquad v = -\frac{1}{x}$

$$= -\frac{(\ln x)^2}{x} + \frac{-2\ln x}{x} + \int \frac{+2}{x^2}dx$$

$$= \frac{-(\ln x)^2}{x} - \frac{2\ln x}{x} - \frac{2}{x}\Big|_1^e$$

$$= \left(\frac{-1}{e} - \frac{2}{e} - \frac{2}{e}\right) - \frac{2}{1} = -\frac{5}{e} - 2$$

$$\int \sin^2 x\cos^2 x\,dx = \int (1-\cos^2 x)\cos^2 x\,dx =$$

$$= \int (\cos^2 x - \cos^4 x)dx$$

$$= \int \left[\frac{\cos 2x+1}{2} - \left(\frac{\cos 2x+1}{2}\right)^2\right]dx$$

$$= \int \left[\frac{\cos 2x}{2} + \frac{1}{2} - \left(\frac{\cos^2 2x + 2\cos 2x + 1}{4}\right)\right]dx$$

$$= \int \left(\frac{\cos 2x}{2} + \frac{1}{2} - \frac{\cos 4x+1}{8} - \frac{\cos 2x}{2} - \frac{1}{4}\right)dx$$

$$= \int \left(\cancel{\frac{\cos 2x}{2}} + \frac{1}{2} - \frac{\cos 4x}{8} - \frac{1}{8} - \cancel{\frac{\cos 2x}{2}} - \frac{1}{4}\right)dx$$

$$= \int \left(\frac{1}{8} - \frac{\cos 4x}{8}\right)dx = \frac{x}{8} - \frac{\sin 4x}{32} + C$$

$$\int \sin^2 x \cos^2 x \, dx$$

$$= \int \left(\frac{1}{2} \cdot 2 \sin x \cos x\right)^2 dx$$

$$= \int \frac{1}{4} \sin^2 2x \, dx$$

1. $\dfrac{3x+1}{(x-1)(x+2)} = \dfrac{A}{x-1} \cdot \dfrac{x+2}{x+2} + \dfrac{B}{x+2} \cdot \dfrac{x-1}{x-1}$

$$\frac{(A+B)(x) + 2A - B}{(x-1)(x+2)}$$

$A+B=3$
$2A-B=1$

2. $\dfrac{2x}{x(x+3)} = \dfrac{A}{x} \cdot \dfrac{x+3}{x+3} + \dfrac{B}{x+3} \cdot \dfrac{x}{x} = \dfrac{\overbrace{(A+B)}^{=2}x + \overbrace{3A}^{=0}}{x(x+3)}$

3. $$\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

4. $$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

5. $$\frac{x^2+11x-1}{x^2(x^2+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+3}$$

6. $$\frac{1}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$$

$$\frac{1}{x^2(x-1)^3(x^2+2)^2} =$$

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2} + \frac{E}{(x-1)^3} + \frac{Fx+G}{(x^2+2)} + \frac{Hx+I}{(x^2+2)^2}$$

$\sin 2x = 2\sin x\cos x$

$$\left.\begin{aligned}\cos 2x &= 2\cos^2 x - 1\\ \cos 2x &= 1 - 2\sin^2 x\end{aligned}\right\}$$

$\sin 8x = \sin 2(4x)$

$= 2\sin 4x\cos 4x$

$\cos^2 x = \dfrac{\cos 2x + 1}{2}$

$\sin^2 x = \dfrac{1 - \cos 2x}{2}$

$\cos^2 12x =$

$= \dfrac{\cos 24x + 1}{2}$

$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1 - \cos^2 x$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\tan x)' = \sec^2 x$

$(\sec x)' = \sec x \tan x$

$(\csc x)' = -\csc x \cot x$

$(\cot x)' = -\csc^2 x$

Log rules

$\ln(x^5) = 5 \ln x$ $\quad \log_a b^p = p \log_a b$

$(\ln x)^5 \neq 5 \ln x$

$\ln(x+1) \neq$

$\ln(x \cdot 2) = \ln x + \ln 2$

$\log_a(MN) = \log_a M + \log_a N$

$\log_a \frac{M}{N} = \log_a M - \log_a N$

$\log(x-1) \neq$

$$\int \sec x \, dx \cdot \frac{\sec x + \tan x}{\sec x + \tan x} = \int \frac{du}{u}$$

$$\int \csc x \, dx \cdot \frac{\csc x + \cot x}{\csc x + \cot x} = \int \frac{-du}{u}$$

$$\int x\sqrt{4+x^2}\, dx \quad , \quad x = 2\tan\theta \Rightarrow \tan\theta = \frac{x}{2}$$

$$dx = 2\sec^2\theta \, d\theta$$

$$\sqrt{4+x^2} = \sqrt{4+(2\tan\theta)^2}$$

$$= \sqrt{4+4\tan^2\theta} = \sqrt{4(1+\tan^2\theta)}$$

$$= \sqrt{4\sec^2\theta} = 2\sec\theta$$

$$= \int (2\tan\theta)\cdot 2\sec\theta \cdot 2\sec^2\theta \, d\theta$$

$$= \int 8\sec^2\theta \cdot \sec\theta\tan\theta \, d\theta \qquad u = \sec\theta$$

$$du = \sec\theta\tan\theta\, d\theta$$

$$= \int 8u^2\, du = \frac{8}{3}u^3 = \frac{8}{3}\sec^3\theta$$

$$= \frac{8}{3}\left(\frac{\sqrt{x^2+4}}{2}\right)^3 = \boxed{\frac{1}{3}(x^2+4)^{3/2} + C}$$