Victoria Gehm

General/Particular solutions

$f'(s) = 6x \quad , \quad f(0) = 8$

$f(s) = 3x^2 + c$ ← general solution — Find the integral of $f'(s)$

$8 = 3(0^2) + c$ — Plug in $f(0)=8$

$8 = 0 + c$

$8 = c$

$\boxed{f(s) = 3x^2 + 8}$ ← particular solution — Plug c back into the original equation

$f''(x) = x^{-3/2} \quad , \quad f'(4) = 2, f(0) = 0$

$f'(x) = -2x^{-1/2} + c$ ← general solution

$2 = -2\left(\frac{1}{\sqrt{4}}\right) + c$ ← plug in $f'(4) = 2$ $\left(4^{-1/2} = \frac{1}{\sqrt{4}}\right)$

$2 = -1 + c$

$3 = c$

$f'(x) = -2x^{-1/2} + 3$

$f(x) = -4x^{1/2} + 3x + c$

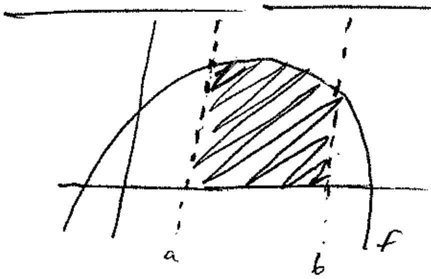
$0 = -4(0) + 3(0) + c$ ← plug in $f(0)=0$

$0 = c$

$f(x) = -4x^{1/2} + 3x + 0$

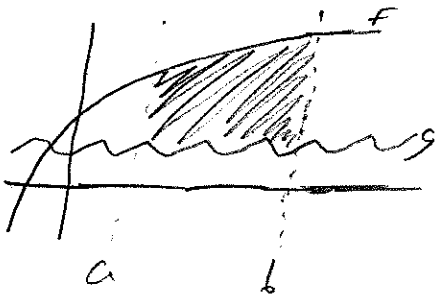
$= \boxed{-4x^{1/2} + 3x}$ ← particular solution

Area of a Region



Area of a region bounded by $f(x)$ & x-axis, between a and b is

$$\int_a^b f(x)\,dx$$



Area of a region between two function $f(x)$ & $g(x)$, between a and b is

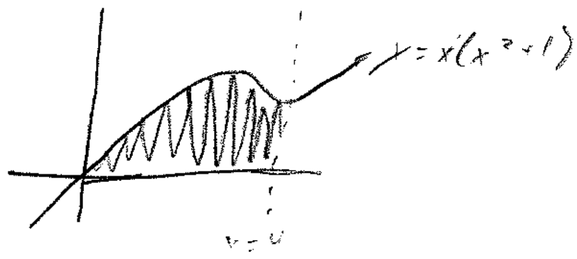
$$\int_a^b \big(f(x) - g(x)\big)\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

Function on top

Function on bottom

Examples:

① Find the area of the region bounded by the graphs of the equations $y = x^3 + x$, $x = 4$, $y = 0$.



$$\int_0^4 (x^3 + x)\,dx$$

$$= \frac{1}{4}x^4 + \frac{1}{2}x^2 \Big|_0^4$$

$$= \frac{1}{4}(4)^4 + \frac{1}{2}(4)^2 - 0$$

Ian Bunker

2/10/17 Riemann Sums + Definite Integrals

$$\sum_{i=1}^{n} f(c_i)\,\Delta x_i \quad , \quad x_{i-1} \le c_i \le x_i$$

$$\frac{b-a}{n} = \Delta x \qquad \lim_{\Delta x \to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x = \int_a^b f(x)\,dx$$

limit as a definite integral, where c_i is any point in the i^{th} subinterval

interval = [0, 4]

$$\lim_{|\Delta x| \to 0} \sum_{i=1}^{n} 5c_i(6-c_i)^2 \Delta x_i =$$

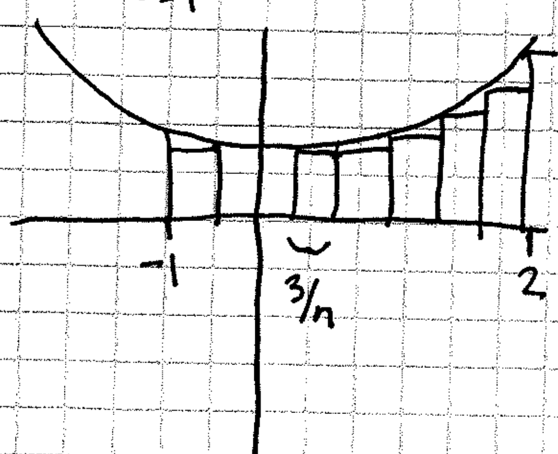
$c_i = x$ $\quad \boxed{\int_0^4 5x(6-x)^2\,dx}$

definite integral as a limit

$$\int_{-1}^{2} (3x^2+2)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(3\left(-1+\tfrac{3}{n}i\right)^2+2\right)\cdot\left(\frac{2-(-1)}{n}\right)$$

height of interval

width interval



$$= \lim_{\Delta x \to 0} \sum_{i=1}^{n} \left[3\left(-1+\Delta x_i\right)^2+2\right]\cdot \Delta x$$

Luke Jastrowski

Mean Value Theorem: If f is continuous on the closed interval $[a,b]$ then there exists a number c in the closed interval $[a,b]$ such that $\int_a^b f(x)\,dx = f(c)(b-a)$

Average Value of a Function on an Interval: If f is integrable on the closed interval $[a,b]$, then the average value f on the interval is $\frac{1}{b-a}\int_a^b f(x)\,dx = f(c) =$ average value of f

Ex: $f(x) = \frac{9}{x^3}$ $[1,3]$

$$\int_1^3 \frac{9}{x^3}\,dx = \frac{9}{c^3}(3-1)$$

$$\int_1^3 9x^{-3}\,dx = \frac{18}{c^3}$$

$$-\frac{9}{2}x^{-2}\Big|_1^3 = \frac{18}{c^3}$$

$$-\frac{9}{2(3)^2} - -\frac{9}{2(1)^2} = \frac{18}{c^3}$$

$$\frac{8}{2} = \frac{18}{c^3}$$

$$4 = \frac{18}{c^3}$$

$$c^3 = \frac{18}{4} = \frac{9}{2}$$

$$c = \sqrt[3]{\frac{9}{2}}$$

Trig substitution

$$\int \frac{10}{x^2\sqrt{25-x^2}}$$

$x = 5\sin\theta \qquad x^2 = 25\sin^2\theta$

$dx = 5\cos d\theta$

step 1: plug trig substitution

$$= \int \frac{10 \cdot 5\cos d\theta}{(5\sin\theta)^2\sqrt{25-(5\sin\theta)^2}}$$

$$= \int \frac{10 \cdot 5\cos\theta \; d\theta}{25\sin^2\theta\sqrt{25(1-\sin^2\theta)}}$$

$\boxed{1-\sin^2\theta = \cos^2\theta}$

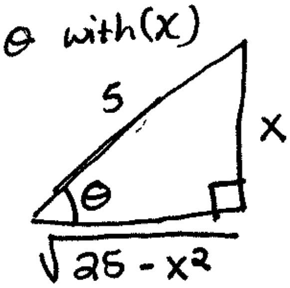
step 2: simplyfy & factor

$$= \int \frac{\overset{2}{\cancel{50}}\,\cancel{\cos\theta}\; d\theta}{\cancel{25}\sin^2\theta \cdot 5\,\cancel{\cos\theta}}.$$

$$= \frac{2}{5}\int \frac{d\theta}{\sin^2\theta} = \frac{2}{5}\int \csc^2\theta \; d\theta$$

$\boxed{\frac{1}{\sin^2\theta} = \csc^2\theta}$

Step 3: use triangle to replace θ with (x)



$x = 5\sin\theta$

$\sin\theta = \frac{x}{5}$

$$= -\frac{2}{5}\cot\theta + C$$

$$= -\frac{2}{5} \cdot \frac{\sqrt{25-x^2}}{x} + C$$

$\boxed{\cot x = \frac{\cos x}{\sin x}}$

[illegible] : [illegible]

Used in certain circumstances where there is an integral of a rational function where the polynomial denominator does not cancel with the numerator.

EX: $\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx$ **STEPS:**

$\frac{5x^2+20x+6}{x(x+1)^2}$ ① factor the denominator completely

$= \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ ② find partial fraction form

- make sum of fractions w/ factors as denominators
- add a fraction for each instance (degree) of factor
  (ex: $(x+1)^2$ is of degree 2, so there is a $\frac{B}{x+1}$ term and a $\frac{C}{(x+1)^2}$)
- numerator is a term/equation one degree lower than its corresponding factor
  (ex: $\frac{A}{x}$; $\frac{Ax+B}{x^2+1}$; $\frac{Ax^2+Bx+C}{x^3+1}$; etc.)

$= \frac{A(x+1)^2}{x(x+1)^2} + \frac{Bx(x+1)}{x(x+1)^2} + \frac{Cx}{x(x+1)^2}$ ③ find common denominator and simplify

$= \frac{A(x^2+2x+1)+Bx^2+Bx+Cx}{x(x+1)^2}$

$= \frac{Ax^2+2Ax+A+Bx^2+Bx+Cx}{x(x+1)^2}$

$= \frac{(A+B)x^2+(2A+B+C)x+A}{x(x+1)^2}$ ④ group based on x-terms

$A+B=5$

$2A+B+C=20$ ⑤ set equal to original equation
- make system of equations and solve

$A=6$

Once solved, $A=6$, $B=-1$, $C=9$

$\int \left(\frac{6}{x} + \frac{-1}{x+1} + \frac{9}{(x+1)^2}\right) dx$ ⑥ substitute into partial fraction form and take integrals

$= 6\int \frac{dx}{x} - \int \frac{dx}{x+1} + 9\int \frac{dx}{(x+1)^2}$

$= 6\ln|x| - \ln|x+1| - \frac{9}{x+1} + C$

⑦ CELEBRATE!

Trina Lin

Riemann Sums and Definite Integrals

- Definite integrals of f gives you point a and b.

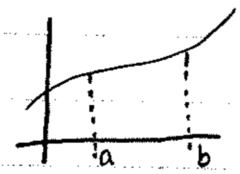
PROPERTIES

- If f(a) is defined (Point A to Point A...)

$$\int_a^a f(x)\,dx = 0$$

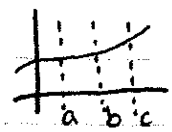
- If f is integral on [a,b] (Point A to Point B)

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$



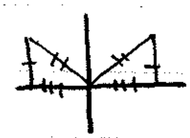
- Concept: Sum of the parts equals the whole (ex. 1+2=3)

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$



- Concept: ab = a·b = a(b)

$$\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$$ **k = constant



- Concept: Sum of the parts equals the whole

$$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

- If $f(x) \geq 0$ on $[a,b]$, $\int_a^b f(x)\,dx \geq 0$
- If $f(x) \leq g(x)$, $\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$

SOLVING FOR AREA

STEP 1: Determine the integral. What is a? What is b?

STEP 2: Determine Δx or dx. $\frac{b-a}{n}$. Example: $\frac{1-(-2)}{n} = \frac{3}{n}$

STEP 3: Determine x. $a + i(\Delta x)$. Example: $-2 + \frac{3}{n}(i)$

STEP 4: Plug it in.

*Follow example below

Evaluate $\int_{-2}^{1} 2x\,dx \Rightarrow$ Step 1: $a = -2 \quad b = 1$

Step 2: $\frac{b-a}{n}$, $\frac{1-(-2)}{n} = \frac{3}{n} = \Delta x$

Step 3: $x = a + i(\Delta x) = -2 + \frac{3}{n}(i) = x$

Step 4: $\lim_{n\to\infty} \sum_{i=1}^{n} f(x)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} 2\left(-2+\frac{3i}{n}\right)\left(\frac{3}{n}\right)$

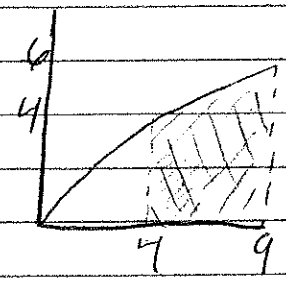
Step 5: [Evaluate] $\lim_{n\to\infty} \left(\frac{3}{n}\right)(2) \sum_{i=1}^{n} \left(-2+\frac{3i}{n}\right) = \lim_{n\to\infty} \frac{6}{n}\left(-2\sum_{i=1}^{n} 1 + \frac{3}{n}\sum_{i=1}^{n} i\right)$

$= \lim_{n\to\infty} \frac{6}{n}\left\{-2n + \frac{3}{n}\left[\frac{n(n+1)}{2}\right]\right\} = \lim_{n\to\infty}\left(-12+9+\frac{9}{n}\right) = \boxed{-3}$

Surface Area

$$S = 2\pi \int_a^b r(x)\sqrt{1+[F'(x)]^2}\,dx$$ ⇒ Formula

$y = 2\sqrt{x}$ $(4, 9)$ - revolve about x Axis



$r(x) = 2\sqrt{x}$, $F'(x) = \frac{1}{\sqrt{x}}$

Answer: $\frac{80\pi}{3}\sqrt{10} - \frac{40\pi}{3}\sqrt{5}$

Arc Length.

By zach (The Best)

- ArcLength of a region refers to the length of a graphed line by measuring the hypotenuses of rectangles that make up the graph.

- You do this by using the equation: $\int_a^b \sqrt{1+(f'(x))^2}$

- In words: Find the integral from a to b of the square root of one plus the Derrivative of the function squared.

- when there are multiple lines subtract the arclength of the line graphically at the bottom from the one on top.

Barrett Purdy

Second Fundamental Theorem of Calculus

If the function f is continuous on an open interval I containing a, then, for every x in the interval the derivative of the integral from a to x is the function. However if it is the derivative of the integral from a to another function then you must also multiply by the derivative of the function in order to account for the chain rule.

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

$$\frac{d}{dx}\left[\int_a^{g(x)} f(t)\,dt\right] = f(g(x)) \cdot g'(x)$$

Examples (a = any constant)

1. $$\frac{d}{dx}\left[\int_a^x t^2\,dt\right] = f(x)$$

$$f(x) = (x)^2 = x^2$$

2. $$\frac{d}{dx}\left[\int_a^{5x^2} t^2\,dt\right] = f(g(x)) \cdot g'(x)$$

$$(5x^2)^2 \cdot 10x$$

$$25x^4 \cdot 10x = 250x^5$$

Savannah Campbell

8.2 Integration By Parts (x^n + cyclic)

Formula:

$\int udv = uv - \int vdu$

Common Integrals Using Integration by Parts:

$\int x^n e^x \, dx$

$\int x^n \cos x \, dx$

$\int x^n \sin x \, dx$

Guidelines for Integration by Parts:

- Let dv = the more complicated part and u = the remaining factors
- Let u = something that has a simpler derivative than its original (example: $(\ln x)' = 1/x$) and dv = the remaining part
- Remember that dv will always include the original equation's dx

Examples:

1. $\int x\, e^x \, dx$

$u = x$	$dv = e^x\, dx$	Choose your u and dv
$du = dx$	$v = e^x$	Take the derivative of u and integrate dv
$uv - \int vdu$		FORMULA
$x\, e^x - \int e^x\, dx$		Plug your u and v into the formula
$x\, e^x - e^x + c$		Integrate $\int$ vdu to solve

2. $\int x^2 \sin x \, dx$

$U = x^2$	$dv = \sin x\, dx$	Choose your u and dv
$Du = 2xdx$	$v = -\cos x$	Take the derivative of u and integrate dv
$uv - \int vdu$		FORMULA
$-x^2\cos x - \int -2x\text{cox}dx$		Plug u and v into your formula
$U = 2x$	$dv = -\cos xdx$	Choose u and v again
$Du = 2dx$	$v = -\sin x$	Take the derivative of u and integrate dv
$-x^2\cos x + 2x\sin x - \int -2\sin xdx$		Plug your u and v into the formula again
$-x^2\cos x + 2x\sin x + 2\cos x + c$		Integrate $\int$ vdu to solve

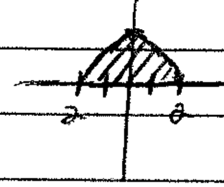
Jacob Little

Using graphs and geometric formulas

Area under a function $= \int_a^b f(x)\,dx$

If f is continuous + nonnegative on the closed interval $[a,b]$, then the area of the region bounded by the graph of f, the x-axis, and the vertical lines $x=a$ and $x=b$.

ex: $\int_1^3 \sqrt{4-x^2}\,dx$, $f(x)=\sqrt{4-x^2}$



The area of the region can be thought of as half the area of a circle of radius 2.

ex: $\int_1^3 4\,dx$, $f(x)=4$

The area of this region can be seen as a rectangle of width 2 and length 4. So, the area will be equal to 4×2

Common Area formula's:

Rectangle/Square $= L \cdot W$

Triangle $= \frac{1}{2}bh$

Circle $= \pi r^2$, Half-circle $= \frac{1}{2}\pi r^2$

$\ln x$ by parts

Nathan Haines

Things to Know

rule #1: always set $\ln x$ as u!
- We don't know how to integrate a natural log, so always set it as u to derive it.

* The derivitive of $\ln x = \frac{1}{x}$

The substitution by parts formula:

$$\int u\,dv = uv - \int v\,du$$

Example with steps:

$\int \frac{(\ln(x))^2}{x^2}$ ← problem

$u = \ln x \quad dv = x^2$
$du = \frac{1}{x} \quad v = \frac{x^3}{3}$ ← pick your u and dv and integrate/derive. ($\ln x$ will always be u)

$\ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x}$ ← apply the formula

$\frac{x^3 \ln x}{3} - \int \frac{x^3}{3x}$ ← multiply your parts

$\frac{x^3 \ln x}{3} - \int \frac{x^2}{3}$ ← simplify the integral (if possible)

$\frac{x^3 \ln x}{3} - \frac{x^3}{9}$ ← integrate the second half

Yay! You did it!

U-Substitution!

Mitchell Patterso

A few IMPORTANT Rules

① The du is <u>never</u> in the bottom
Wait/that's it...

There must be a function & a function that is exactly or closely related to its differential.

① A.K.A. $\int \underbrace{(x^3+5)}_{u} \underbrace{2x^2 dx}_{\text{Differential of } u}$

$u = x^3+5$
$du = 2x^2 dx$

After substituting for the u & du, the above integral is equivalent to:

② $\int u \, du$ - Now solve it like a regular problem! Yay!

$\int u \, du = \frac{1}{2}u^2$ ③ - BUT WAIT!
- Remember to <u>SUBSTITUTE THE U BACK IN!</u> :)

④ If $u = x^3+5$, then $\frac{1}{2}u^2$

$$\boxed{= \frac{1}{2}(x^3+5)^2}$$

← The answer!

- But they won't always be so nice...

EXAMPLE A

$$\int \frac{2x-4}{x^2+4x+13}\,dx$$

Sometimes completing the square might be necessary.

Notice: The derivative of $x^2+4x+13 = 2x+4$, so:

① Add zero $(2x+4-8 = 2x-4)$

$$\int \frac{2x+4}{x^2+4x+13} - \int \frac{8}{x^2+4x+13}$$

② $\displaystyle\int \frac{2x+4}{x^2+4x+13}$ $u = x^2+4x+13$, $du = 2x+4\,dx$

$$\int \frac{du}{u} = \ln|x^2+4x+13|$$

③ $\displaystyle 8\int \frac{1}{x^2+4x+4-4+13}$

$$= 8\int \frac{1}{x^2+4x+4+9} = 8\int \frac{1}{(x+2)^2+9}$$

$$\boxed{= \ln|x^2+4x+3| - \frac{8}{3}\arctan\left(\frac{x+2}{3}\right) + C}\ *$$

*Use arctan, $\frac{1}{u^2+a^2} = \frac{1}{a}\arctan\frac{u}{a}$

where $u = x+2$

EXAMPLE B

$$\int \tan 6\theta\, d\theta$$

Sometimes the du needs to be played with

$$= \int \frac{\sin 6\theta}{\cos 6\theta}\, d\theta \qquad u = \cos 6\theta,\quad du = -6\sin\theta\, d\theta$$

The du isn't exactly what we need, so

$du = -6\sin\theta\, d\theta$

$-du = 6\sin\theta\, d\theta$

$-\frac{1}{6}du = \sin\theta\, d\theta$

Perfect!

$$\int -\frac{1}{6}\cdot\frac{du}{u} = -\frac{1}{6}\int\frac{du}{u}$$

$$\boxed{= -\frac{1}{6}\ln|\cos 6\theta| + C}$$

Simple!

Hannah Lee

Integral Study Guide - Volume of Solids of Revolution

- From what I have studied so far, there are 4 types of V.O.S.R problems.
  1) revolving around x-axis
  2) revolving around y-axis
  3) revolving around x= something
  4) revolving around y= something

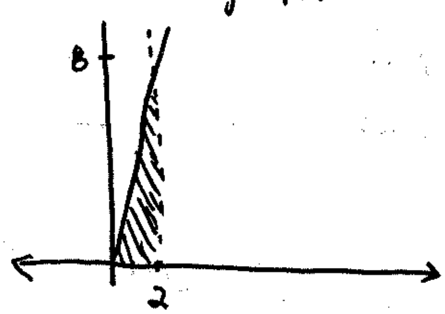
However, my ultimate 6 step approach can solve all 4 types!

- First, draw the graph.
- Second, determine where to revolve around. dx or dy?
- Third, draw the volume solid (optional if you hate drawing)
- Fourth, determine the radius
- Fifth, plug in to the formula
- Sixth, evaluate if Ms. Brewer or instruction tells you to

< Example 1-1 >

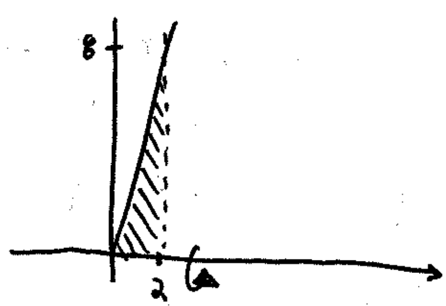
$y = 2x^2$, $y = 0$, $x = 2$, revolve about the line x-axis

▲ Draw the graph ◀



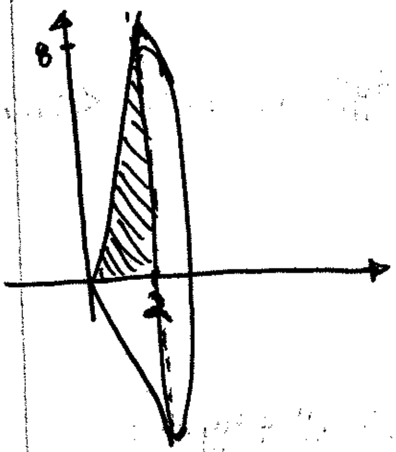
$\Rightarrow$

▲ Determine the line ◀



$\Rightarrow$

▲ Draw the solid ▲ ▲ Determine the radius ▲



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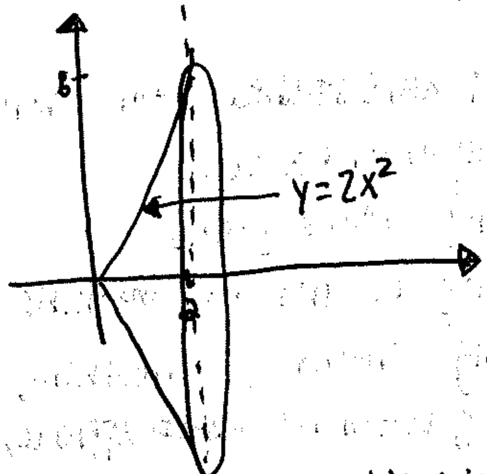
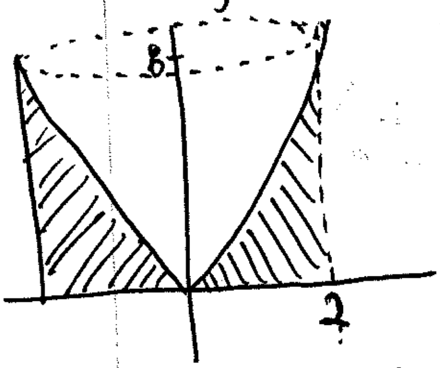


- We can see that the coordinates for x-axis problem is found at x-axis therefore, $\int_0^2 \pi (2x^2)^2 \, dx$.

<Example 1-2>

Let's do the same problem but revolve around y-axis. This is a little different than x-axis problem but it's still fairly simple.



- We can obviously see that the solid formed a cylinder. However, it is hollow inside. In this case, the volume of solid changes to

volume of outer solid − volume of inside.

$$V = 32\pi - \int_0^8 \pi y/2 \, dy$$

equals to

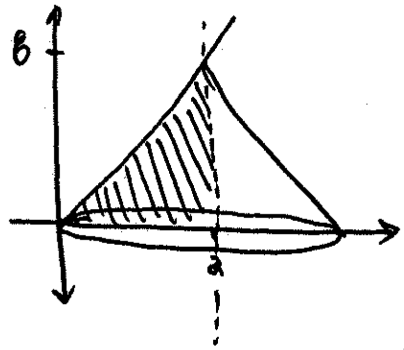
$$V = [\pi (2)^2 (8)] - \int_0^8 \pi y/2 \, dy.$$

Why not $2x^2$? because solid is revolved around y-axis the radius has to change to " to y".

coordinates for y-axis!

<Example 1-3>

revolved around $x=2$



$$V = \int_0^8 \pi \left(2 - \sqrt{\frac{y}{2}}\right)^2 dy$$

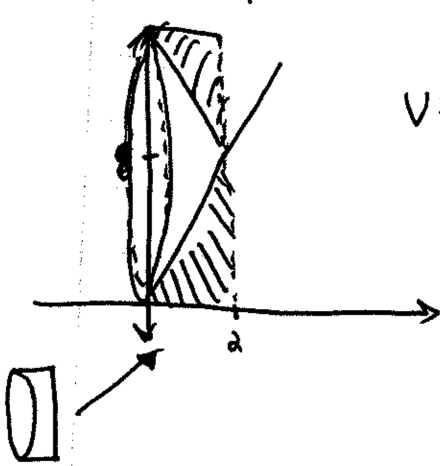
why subtract 2?

in V.O.S.R, radius means the distance from origin. therefore, since the radius is $x=$ something rather than x-axis or y-axis we have to subtract it.

<Example 1-4> revolve around $x=8$

V outer cylinder - V bowl

$$V = \pi(8)^2 2 - \int_0^2 \pi(8-2x)^2 \, dx$$



Hanna Bobinger

Final Exam Study Guide

Position, Velocity, and Acceleration - Linear Motion

I. Position

- the derivative of a function can represent the rate of change of one variable with respect to another → rate of change can describe the motion of an object moving in a straight line
  ↳ the line of motion is represented by a horizontal or vertical line with a designated origin, and the position function (s) gives the position (relative to the origin) of an object as a function of time

- $\Delta s = s(t + \Delta t) - s(t)$

where Δs = (final position - initial position) = change in distance
s = position at time t
t = time
Δt = (time at final position - time at initial position) = change in time
$s(t)$ = position at time t

Rate = distance / time

II. Average Velocity of a Falling Object

$$\text{average velocity} = \frac{\text{change in distance}}{\text{change in time}} = \frac{\Delta s}{\Delta t}$$

Steps to Solve:

Step 1) Given the position function and the time interval, solve for the final and initial position.

Step 2) Subtract the initial position from the final position to get Δs.

Step 3) Subtract initial time from the final time to get Δt.

Step 4) Plug in the Δs and Δt values into the average velocity formula to obtain your answer.

Example: Find the average velocity over the time interval [1,2]
Position function: $s = -16t^2 + 100$ where s is measured in feet and t is measured in seconds

Step 1) initial position: $s(1) = -16(1)^2 + 100 = 84$ ft
final position: $s(2) = -16(2)^2 + 100 = 36$ ft
Step 2) $\Delta s = 36 - 84 = -48$ ft
Step 3) $\Delta t = 2 - 1 = 1$ second
Step 4) average velocity $= \frac{-48 \text{ ft}}{1 \text{ sec}} = \boxed{-48 \text{ ft/sec}}$

* Note: The negative average velocity indicates that the object is moving downward *

III. (Instantaneous Velocity)

- the average velocity is the slope of the secant line
- the instantaneous velocity is the slope of the tangent line
  - you can approximate the slope of the tangent line by calculating the slope of the secant line
  - you can also approximate the instantaneous velocity at time t by calculating the average velocity over a small time interval $[t, t+\Delta t]$, and by taking the limit as Δt approaches zero, you can obtain the velocity at t

$$\boxed{v(t) = \lim_{\Delta t \to 0} \frac{s(t+\Delta t) - s(t)}{\Delta t} = s'(t)}$$

- The position function of a free-falling object is

$$\boxed{s(t) = \frac{1}{2}gt^2 + v_0 t + s_0}$$

s_0 = initial height
v_0 = initial velocity
g = accl. due to gravity

and the velocity function is the derivative of the position function

- on Earth, $g \approx 9.8$ m/s^2 or -32 ft/s/s

Marina Bolinger

• velocity can be positive, negative, or zero
• Speed = |velocity|
• when $v=0$, the object is at its maximum height

Steps to Solve:

1) To find the instantaneous velocity at time t given the position function, find the derivative of the position function. This is the velocity function.
2) Plug in t to the velocity function to find the instantaneous velocity.

Example: Find the velocity at time $t=1.5$ seconds.
Position function: $-16t^2+16t+32$

Step 1) $s'(t)=v(t)=-32t+16$

Step 2) $v(1.5)=-32(1.5)+16=\boxed{-32\ ft/sec}$

* Note: velocity is positive when an object is rising and negative when an object is falling. *

IV. Acceleration

• you can obtain an acceleration function by differentiating a velocity function or differentiating a position function twice

• Position function: $s(t)$
• Velocity function: $v(t)=s'(t)$
• Acceleration function: $a(t)=v'(t)=s''(t)$

• In other words, the acceleration function $a(t)$ is the second derivative of $s(t)$, which is an example of a higher-order derivative, and you can define derivatives of an positive integer order:

First derivative	y'	$f'(x)$
Second derivative	y''	$f''(x)$
Third derivative	y'''	$f'''(x)$
Fourth derivative	$y^{(4)}$	$f^{(4)}(x)$
⋮		
n^{th} derivative	$y^{(n)}$	$f^{(n)}(x)$

Steps to Solve:

1) a) To find the acceleration at time t given the position function, differentiate the position function twice.

b) To find the acceleration at time t given the velocity function, differentiate the velocity function once.

2) Plug in t to the acceleration function to obtain the value for the acceleration at time t.

Example: Find the acceleration of a car starting from rest at t = 5 sec

velocity function: $v(t) = \frac{100}{2t+15}$, where v is measured in ft/sec

Step 1) $v(t) = \frac{100}{2t} + \frac{100}{15} = 50t^{-1} + \frac{20}{3}$

$v'(t) = -50t^{-2} = a(t)$

Step 2) $a(t) = -50(5)^{-2} = \boxed{-2 \text{ ft/sec}^2}$

Helen Kim

Integrals of Trigonometric Functions

- Derivative Rules

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\sec x = \sec x \tan x$

$\frac{d}{dx}\csc x = -\csc x \cot x$

- Antiderivative Rules

$\int \cos x\,dx = \sin x + c$

$\int \sin x\,dx = -\cos x + c$

$\int \sec^2 x\,dx = \tan x + c$

$\int \csc^2 x\,dx = -\cot x + c$

$\int (\sec x \tan x)\,dx = \sec x + c$

$\int (\csc x \cot x)\,dx = -\csc x + c$

$\int \tan x\,dx = -\ln|\cos x| + c$

$\int \cot x\,dx = \ln|\sin x| + c$

$\int \sec x\,dx = \ln|\sec x + \tan x| + c$

$\int \csc x\,dx = -\ln|\csc x + \cot x| + c$

- $$\int \frac{dx}{\sqrt{a^2-u^2}} = \arcsin \frac{x}{a} + c$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan \frac{x}{a} + c$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\operatorname{arcsec} \frac{|x|}{a} + c$$

- Double Angle Identities

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

Power-Reducing Formulas.

$\sin^2 x = \dfrac{1 - \cos 2x}{2}$

$\cos^2 x = \dfrac{1 + \cos 2x}{2}$

$\tan^2 x = \dfrac{1 - \cos 2x}{1 + \cos 2x}$

Luke Yarbrough

Integration by Parts:

2 cyclics

$$\int e^\theta \cos\theta \, d\theta$$ $u = \cos\theta$, $dv = e^\theta d\theta$, $du = -\sin\theta \, d\theta$, $v = e^\theta$ — Start with integration by parts

$$= e^\theta \cos\theta \underset{(+)}{-} \left(-\int e^\theta \sin\theta \, d\theta\right)$$ $u = \sin\theta$, $dv = e^\theta d\theta$, $du = \cos\theta \, d\theta$, $v = e^\theta$ — ~~Do it~~ again

$$= e^\theta \cos\theta + \left(e^\theta \sin\theta - \int e^\theta \cos\theta \, d\theta\right)$$

$$\int e^\theta \cos\theta \, d\theta = e^\theta \cos\theta + e^\theta \sin\theta - \cancel{\int e^\theta \cos\theta \, d\theta}$$
$$+\int e^\theta \cos\theta \, d\theta \qquad +\cancel{\int e^\theta \cos\theta \, d\theta}$$

Remember that these are equivalent to each other

$$= \frac{2\int e^\theta \cos\theta \, d\theta}{2} = \frac{e^\theta \cos\theta + e^\theta \sin\theta}{2}$$ Then, simple algebra

Simplify

$$\checkmark = \int e^\theta \cos\theta \, d\theta = \frac{e^\theta(\cos\theta + \sin\theta)}{2} + C$$ ✓ Don't forget the constant

$$\int e^{3\theta} \sin 2\theta \, d\theta$$

So, you know that you'll have to do two integration by parts. But, how do you know your u and dv?

$\boxed{u = \text{trig function} \qquad dv = \text{exponential function}}$ ← usually, not always

$$\int e^{3\theta} \sin 2\theta \, d\theta$$ $u = \sin 2\theta$, $dv = e^{3\theta}$, $du = 2\cos 2\theta \, d\theta$, $v = \frac{1}{3} e^{3\theta}$

$$= \frac{e^{3\theta} \sin 2\theta}{3} - \int \frac{e^{3\theta}\, 2\cos 2\theta \, d\theta}{3}$$

$$= \frac{e^{3\theta} \sin 2\theta}{3} - \frac{2}{3}\int e^{3\theta} \cos 2\theta \, d\theta$$ $u = \cos 2\theta$, $dv = e^{3\theta}$, $du = \cancel{-2\sin 2\theta \, d\theta}$, $v = \frac{1}{3} e^{3\theta}$

$$= \frac{e^{3\theta}}{3} \sin 2\theta - \frac{2}{3}\left(\frac{e^{3\theta}}{3} \cos 2\theta + \frac{2}{3}\int e^{3\theta} \sin 2\theta \, d\theta\right)$$ $\frac{4}{9} + \frac{9}{9}$

$$\int e^{3} \sin 2\theta \, d\theta = \frac{e^{3\theta} \sin 2\theta}{3} - \frac{2e^{3\theta} \cos 2\theta}{9} - \frac{4}{9}\int e^{3\theta} \sin 2\theta \, d\theta$$ $\frac{13}{9}$

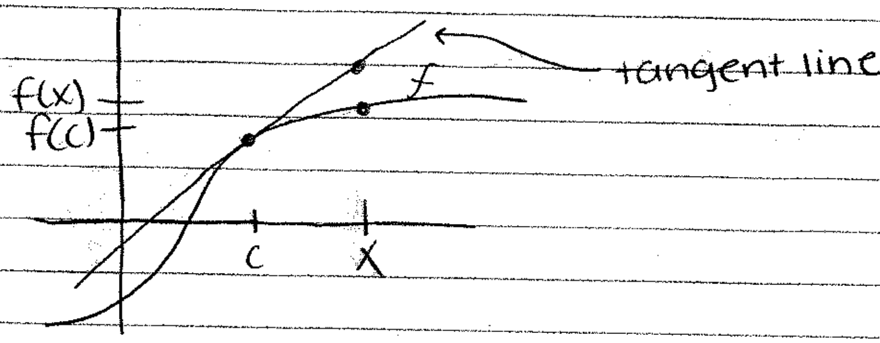
$$\frac{\frac{13}{9}\int e^{3\theta}\sin 2\theta\,d\theta}{13/9} = \frac{\frac{e^{3\theta}\sin 2\theta}{9} - \frac{2e^{3\theta}\cos 2\theta}{9}}{13/9}$$

$$= \int e^{3\theta}\sin 2\theta\,d\theta = \frac{9e^{3\theta}\sin 2\theta}{13(9)} - \frac{18e^{3\theta}\cos 2\theta}{13(9)}$$

$$\boxed{= \int e^{3\theta}\sin 2\theta\,d\theta = \frac{9e^{3\theta}}{117}\left(\sin^2\theta - 2\cos^2\theta\right) + C}$$

{Renata}

Differentials & tangent approximations



if f is differential at c,

eq. tangent line → $y - f(c) = f'(c)(x-c)$

$$y = f(c) + f'(c)(x-c)$$

find the differential dy.

12. $y = 3x^{2/3}$

$dy = 2x^{-1/3}\,dx$

14. $y = \sqrt{9-x^2}$

$dy = \frac{1}{2}(9-x^2)^{-1/2}(-2x)\,dx$

tangent approx.

*derivation ?

$\Delta y = f(c+\Delta x) - f(c)$

$dy = f'(x)\,dx$

$\Rightarrow \Delta y \approx dy$ — set them equal to each other

Use differentials to approximate $\sqrt[3]{26}$

equation to use → $\boxed{f(c+\Delta x) \approx f(c) + f'(c)\,dx \;*}$

$\underbrace{f(x) = \sqrt[3]{x}}_{\text{function}}$ $\quad \underbrace{c = 27}_{\text{"pretty" number you can easily evaluate}}$ $\quad \underbrace{\Delta x = dx = -1}_{\text{what you add to your } c \text{ to get your original value}}$

[ex: 27 + (-1) = 26]

① take derivative of your function:

$f(x) = \sqrt[3]{x} = x^{1/3}$

$f'(x) = \frac{1}{3}x^{-2/3} = \boxed{\frac{1}{3(\sqrt[3]{x})^2}}$

② find f'(c) by plugging c into f'(x)

$f'(27) = \frac{1}{3(\sqrt[3]{27})^2} = \boxed{\frac{1}{27}}$

③ find f(c):

$f(x) = \sqrt[3]{x}$

$f(27) = \sqrt[3]{27} = \boxed{3}$

now plug everything into equation:

$$f(c+\Delta x) = f(c) + f'(c)dx$$

$$f(26) = 3 + \frac{1}{27}(-1)$$

$$f(26) = \left(3 \cdot \frac{27}{27}\right) - \frac{1}{27} \rightarrow f(26) = \frac{81}{27} - \frac{1}{27} = \boxed{\frac{80}{27}}$$

Inverse Trig + Completing The Square

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$$

$$\frac{d}{dx}[\text{arcsec}] = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan \frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\text{arcsec}\frac{|u|}{a} + C$$

Completing the Square:

1. $ax^2 + bx + C$
2. $a\left(x^2 + \frac{b}{a}x\right) + C$
3. $a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + C - \frac{b^2}{4a}$
4. $a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$

You can use completing the square in order to get a denominator like one of the ones in the inverse trig formulas.

Inverse Trig & Completing the Square

P.2

<u>example 1</u>

$$\int \frac{dx}{x^2+2x+5}$$

1. Take the denominator, complete the square

x^2+2x+5

↳ $2/1 = 1$, $1^2 = 1$, so we want this to look like: x^2+2x+1.

We can do this like this: $x^2+2x+\underbrace{1+4}_{5}$

$$= \int \frac{dx}{x^2+2x+1+4} = \int \frac{dx}{(x+1)^2+4} = \int \frac{dx}{(x+1)^2+2^2}$$

recall: $\int \frac{dx}{a^2+u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

So, applying this we get: $\frac{1}{2} \arctan \frac{x+1}{2} + C$

Note: $x+1$ is not "a!" a must be a constant, and is therefore 2

Inverse Trig + Completing The Square

<u>example 2</u>

p. 3

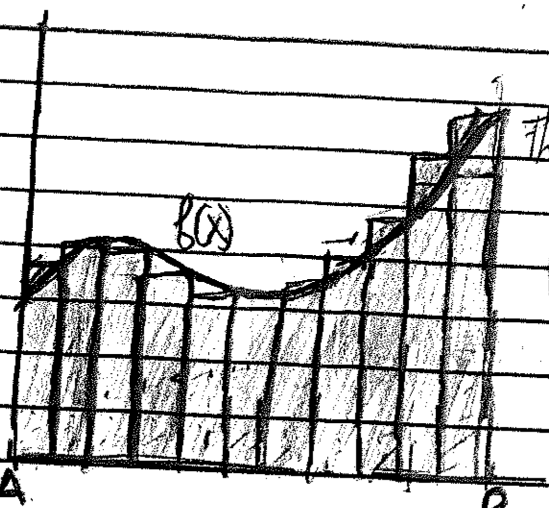
$$\int \frac{dx}{\sqrt{21-4x-x^2}} \longrightarrow 21-4x-x^2 = 21-[x^2+4x]$$
$$= 21+4-[x^2+4x+4]$$
$$= 25-(x+2)^2$$

$$\int \frac{dx}{\sqrt{21-4x-x^2}} = \int \frac{dx}{\sqrt{25-(x+2)^2}} = \int \frac{dx}{\sqrt{5^2-(x+2)^2}}$$

recall $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + C$,

$$\int \frac{dx}{\sqrt{5^2-(x+2)^2}} = \arcsin \frac{x+2}{5} + C$$

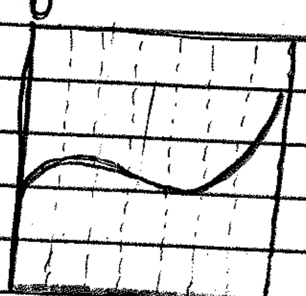
Simon Peoples

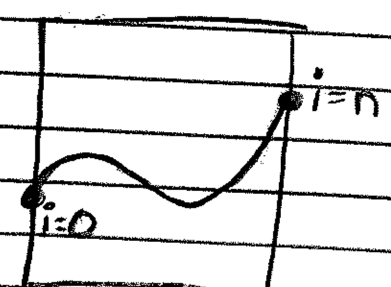


The area under the ln f(x) is approximated from point A to point B

$$\sum_{1=i}^{n=?} f\left(a+\frac{B-A}{n}i\right)\cdot\frac{B-A}{n}$$

- this reads: add up eleven ($n=11$) rectangles defined by height = $f\left(a+\frac{B-A}{n}\right)$ and width = $\left(\frac{B-A}{n}\right)$

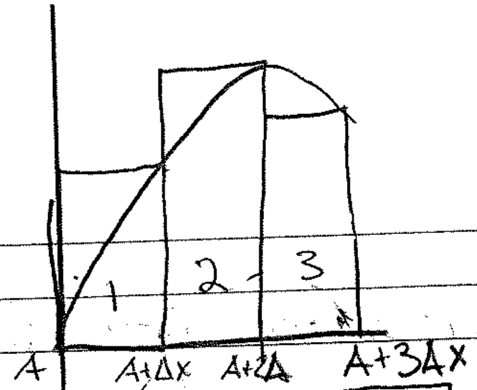




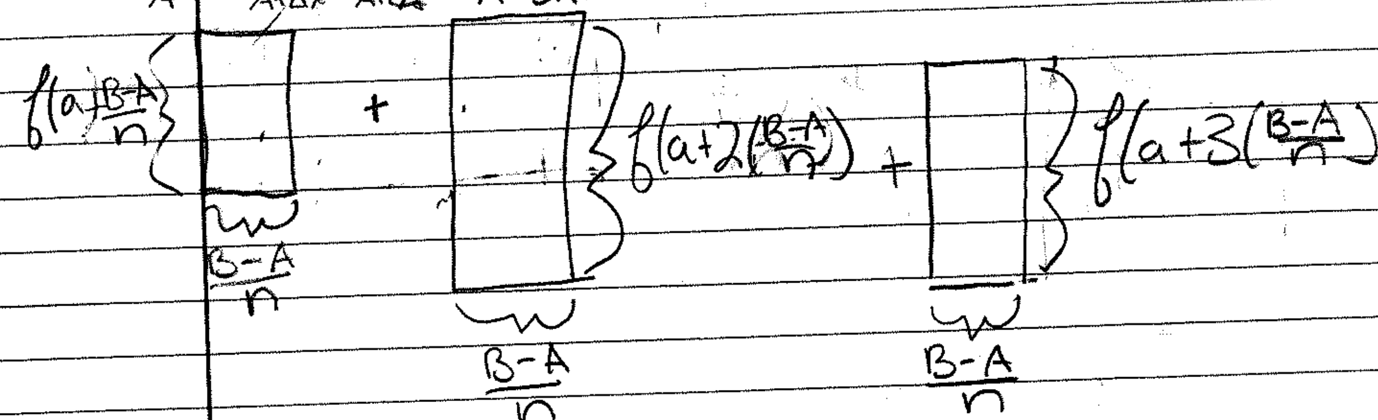
If I divide the graph from A to B evenly in n piece each of the rectangular-ish shapes will have a width of $\frac{B-A}{n}$

to get the heights of these "rectangles" you use $f\left(a+\frac{B-A}{n}i\right)$. You can kinda view it as $f(x+\Delta x)$. so A if your original x was 1 but the value you want is $\Delta x = 2$ away

$f(1+2) = f(3)$

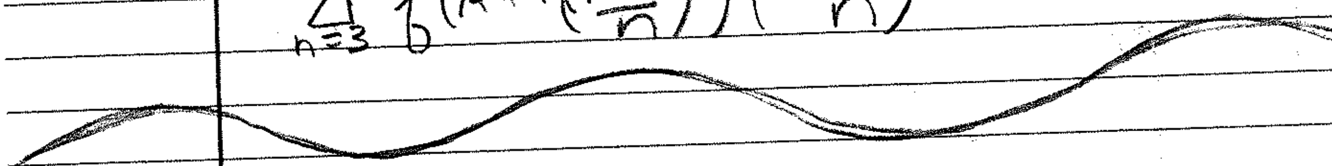


$i\ \Delta x = \frac{B-A}{n}$



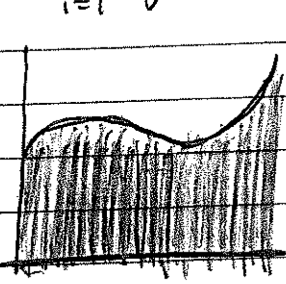
so this would look like

$$\sum_{n=3}^{i=1} f\left(A+i\left(\frac{B-A}{n}\right)\right)\left(\frac{B-A}{n}\right)$$

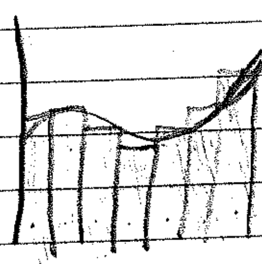


- The graph/equations above have been approximations to find the real area you have to make $n=\infty$. This makes it so you are "aproximating" using infinity rectangles with height $f\left(a+\left(\frac{B-A}{n}\right)i\right)$ and width $\frac{B-A}{n}$

$$\lim_{n\to\infty} \sum_{i=1}^{n=\infty} f\left(a+\left(\frac{B-A}{n}\right)i\right)\left(\frac{B-A}{n}\right)$$



vs.

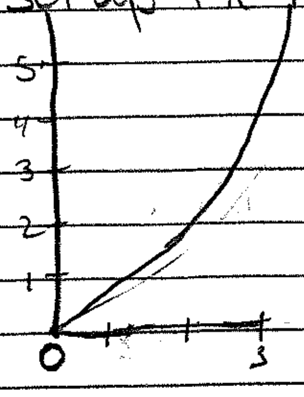


$\lim_{n\to\infty}$

$n=7$

Simmen
Peoples

set up the notation for this graph $f(x) = x^2$ as
n approaches ∞



$$\lim_{n \to \infty} \sum_{i=1}^{n} \left(0 + \frac{3}{n} i\right)^2 \left(\frac{3}{n}\right)$$

to solve these you will need to try and manipulate the notations into simple series like

$$\sum c \;,\; \sum i \;,\; \sum i^2 \;,\; \sum i^3$$

$$\sum_{i=1}^{n} c = cn \;;\; c = \text{constant}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

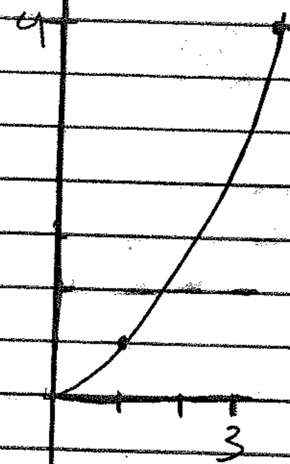
$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

you can achieve this by using rules like:

$$\sum a i^x = a \cdot \sum i^x$$

$$\sum \frac{27}{n^3} i^2 = \frac{27}{n^3} \sum i^2 = \frac{27}{n^3} \left(\frac{n(n+1)(2n+1)}{6}\right)$$

The rest is algebra good luck



$A = 0$
$B = 3$
$f(x) = x^3$

set up the sigma notation and solve

Answer + work

$$\lim_{n\to\infty} \sum_{i=1}^{n=\infty} = \left(0 + \frac{3}{n} i\right)^3 \left(\frac{3}{n}\right) = \sum \frac{81}{n^4} i^3 = \frac{81}{n^4} \sum i^3 =$$

$$\frac{81}{n^4}\left(\frac{n^2(n+1)^2}{4}\right) = \frac{81}{n^4}\left(\frac{n^4 + 2n^3 + n^2}{4}\right) =$$

$$\frac{81}{4} + \frac{2}{4n} + \frac{1}{n^2} = \frac{81}{4} \text{ as } n \to \infty \left(\frac{\mathbb{R}}{\infty} = 0\right)$$

you can check your work with intergrals

$$\int_0^3 x^3 = \frac{1}{4}x^4 \Big|_0^3 = \frac{1}{4}(81) - \frac{1}{4}(0) = \frac{81}{4}$$