TRICOLORABILITY

We will say that a **strand** in a projection of a link is a piece of the link that goes from one undercrossing to another with only overcrossings in between. We will say that a projection of a knot or link is **tricolorable** if each of the strands in the projection can be colored one of three different colors, so that at each crossing, either three different colors come together or all the same color comes together. In order that a projection be tricolorable, we further require that at least two of the colors are used. Figure 1.41 shows that these two projections of the trefoil knot are tricolorable (using white, gray, and black as the colors).

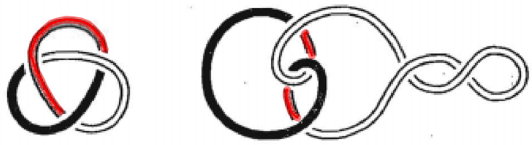


Figure 1.41 The trefoil is tricolorable.

In the first tricoloration, three different colors come together at each crossing, whereas in the second tricoloration, some of the crossings have only one color occurring. But none of the crossings in either picture have exactly two colors occurring, so these are valid tricolorations.

Exercise 1.21 Determine which of the projections of the three six-crossing knots 6_1, 6_2, and 6_3 in Figure 1.42 are tricolorable.

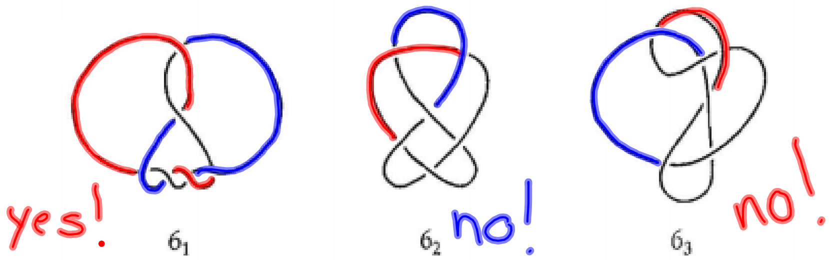
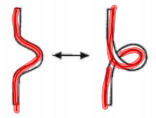


Figure 1.42 Projections of 6_1, 6_2, and 6_3.



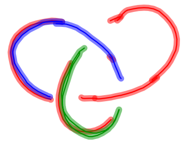


Figure 1.44 Type I moves preserve tricolorability.

class example

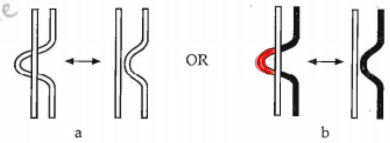
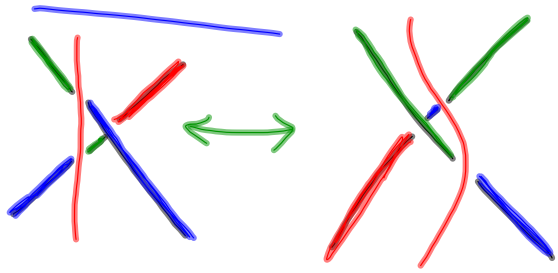


Figure 1.45 Type II moves preserve tricolorability.

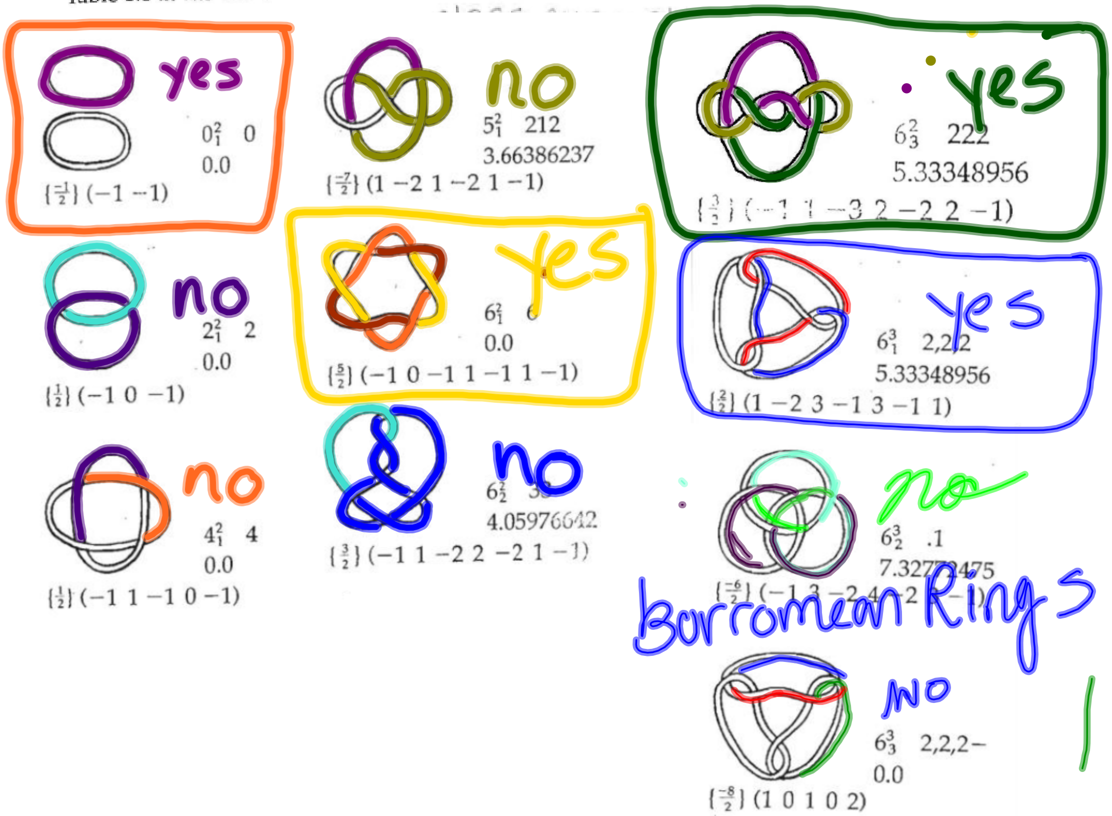
Exercise 1.23 Show that the Type III Reidemeister move preserves tricolorability. (There are several cases to check.) assigned exercise

6 cases



other cases will include single color at crossing, all crossings, etc.

Exercise 1.30 Determine which of the links of six or fewer crossings in Table 1.1 at the end of the book are and are not tricolorable.



For your knot or link, determine:

1. is it a knot or a link? if it is a link, how many components does it have?

2. how many crossings are there?

3. if it is a link, what is the linking number?

4. is it tricolorable?

On a 8.5" x 11" page, include colored knot (tricolored if tricolorable, colored any way you like if not), and above information.

Exercises to work from packet: 1.11, 1.15, 1.17, 1.22, 1.23, 1.24, 1.31

Exercise Hints/Explanation:

1.11 - untangle the knot using Reidemeister moves; clearly indicate which move you are using to get from one step to the next, and perform only one move at a time; multiple type-I or type-II moves may be performed at the same time only if they are obvious and on different parts of the knot (you must indicate type-II x2, e.g.)

1.15 - we computed the linking number for one orientation in class; for homework, reverse the direction on one component and recompute it.

1.17 - we computed the linking number for the link on the left in class; compute it again using a different orientation for one of the components; when computing linking number for the link on the right, remember not to count a crossing between a component and itself

1.22 - we know that the knot is tricolorable, you just have to find such a coloring scheme

1.23 - there are 6 cases to check; we showed one such case in class (3 colors at all crossings, strand moving on top of crossing); other cases will be involving a single color at one crossing and 3 colors at others, and a single color at all crossings; for each, you should include above and below (i.e. the strand moving across the crossing on top of it, and the strand moving across the crossing below it)

1.24 - you must show colorings of each of the 7-crossing knots 7_1-7_7 on pages 280-281 and state which are tricolorable and which are not

1.31 - we know that this link is tricolorable, you just have to find such a coloring scheme