Visual Mathematics - turnitin
Class ID: 9012421

Fractals:
Essay: Fri. 11/14

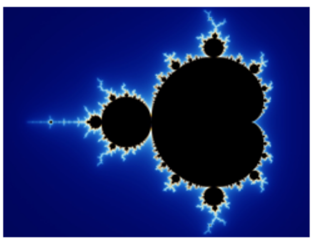
Problem Set: Wed. 11/12

# FRACTALS

**Fractals:**

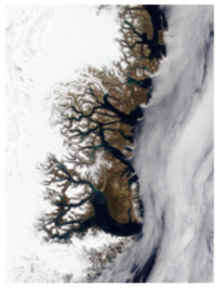
- are self-similar (at least approximately), i.e. have the rescaling property (when you zoom in on a piece it looks like the whole)
- have fine structure on arbitrarily small scales
- often have simple, recursive definitions

Mandelbrot Set

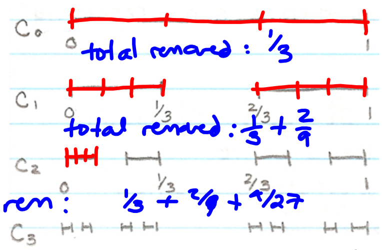


Romanesco broccoli

Coastline of Norway



**The Cantor Middle-Third Set:**




$2^0$ 1 segment of length $1 = \left(\frac{1}{3}\right)^0$

$2^1$ 2 segments of length $\frac{1}{3} = \left(\frac{1}{3}\right)^1$

$2^2$ 4 segments of length $\frac{1}{9} = \left(\frac{1}{3}\right)^2$

$2^3$ 8 segments of length $\frac{1}{27} = \left(\frac{1}{3}\right)^3$

$C_n$ consists of $2^n$ segments, each of length $\left(\frac{1}{3}\right)^n$

To obtain $C_{n+1}$ from $C_n$, we remove the middle third of each interval in $C_n$. The Cantor set $C$ is the intersection of all $C_n$. $C$ is a fractal.

$2^n \cdot \left(\frac{1}{3}\right)^n = \left(\frac{2}{3}\right)^n$

$C_n$ consists of $2^n$ closed intervals of length $\frac{1}{3^n}$. The total length of $C_n$ is $\left(\frac{2}{3}\right)^n$, which approaches 0 as $n$ approaches $\infty$. Hence the "length" of $C$ is 0.

Another way to state this is that the length of $C_{n+1} = \frac{2}{3} \cdot \textit{length of } C_n$ . Given this recursive definition, again we have that the length of $C_n$ is , which approaches 0 as $n$ approaches $\infty$.

The total length removed from the interval $[0\,,1]$ in the construction of the Cantor set is

geometric series w/ common ratio $2/3$

$$\frac{1}{3}+\frac{2}{9}+\frac{4}{27}+\cdots=\sum_{n=0}^{\infty}\frac{2^n}{3^{n+1}}=\sum_{n=0}^{\infty}\frac{1}{3}\left(\frac{2}{3}\right)^n=\frac{\frac{1}{3}}{1-\frac{2}{3}}=\frac{\frac{1}{3}}{\frac{1}{3}}=\boxed{1}$$

$\frac{2^0}{3^1}+\frac{2^1}{3^2}+\frac{2^2}{3^3}$   $3^{n+1} = 3 \cdot 3^n$

Hence we have a set from which its entire length has been removed. Yet there are still infinitely many points left in the set. Which points are they?

Note: the sum of an infinite geometric series with common ratio less than 1 in absolute value is equal to $\boxed{S_\infty = \frac{a_1}{1-r}}$, where $a_1$ is the first term and $r$ is the common ratio.