Turnitin.com Class ID: 9012421
Mon. 11/10 finish fractal lecture
Tues. 11/11 fractal problem set work day
Wed. 11/12 fractal problem set due; fractal essay work day
Fri. 11/14 fractal essay due -- **PRESENTERS?**
Kinsey - Art ; Lexi - Nature

If you haven't already, start reading articles in my Google Drive folder, search the Alabama Virtual Library, and peruse textbooks in S201 and the ASMS Library!

Textbooks/journal articles available for this topic in S201 for use in Math Lab:

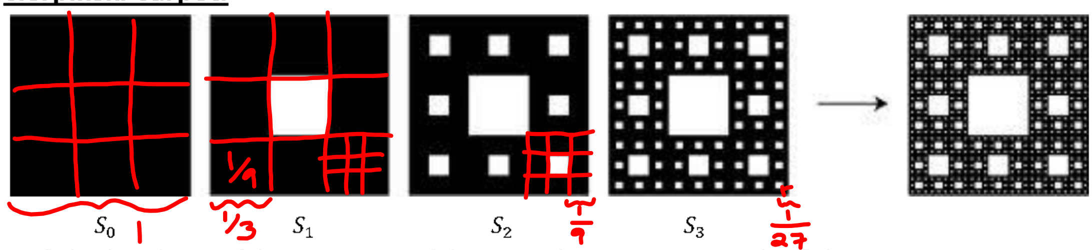
1. Barnsley, Michael. *Fractals Everywhere*. San Diego: Academic Press, 1988.
2. Devaney, Robert. *Chaos, Fractals, and Dynamics: Computer Experiments in Mathematics*. Menlo Park: Addison-Wesley, 1990.
3. Devaney, Robert and Keen, Linda, ed. *Chaos and Fractals: The Mathematics Behind the Computer Graphics*. Proceedings of Symposia in Applied Mathematics, Vol 39. Providence: American Mathematical Society, 1989.
4. Frantz, Marc and Crannell, Annalisa. *Viewpoints: Mathematical Perspective and Fractal Geometry in Art*. Princeton : Princeton University Press, 2011.
5. Peitgen, H.-O. and Richter, P.H. *The Beauty of Fractals: Images of Complex Dynamical Systems*. Berlin: Springer-Verlag, 1986.
6. Peitgen, Heinz-Otto and Saupe, Deitmar, ed. *The Science of Fractal Images*. New York: Springer-Verlag, 1988.
7. Schroeder, Manfred. *Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise*. New York: W.H. Freeman, 1991.
8. Segerman, Henry. "Fractal Graphs by Iterated Substitution," *Journal of Mathematics and the Arts*, Vol. 5, No. 2, June 2011, pp. 51-70.

In the ASMS Library:

1. Mandelbrot, Benoit. *The Fractal Geometry of Nature*. New York: W.H. Freeman, 1983.

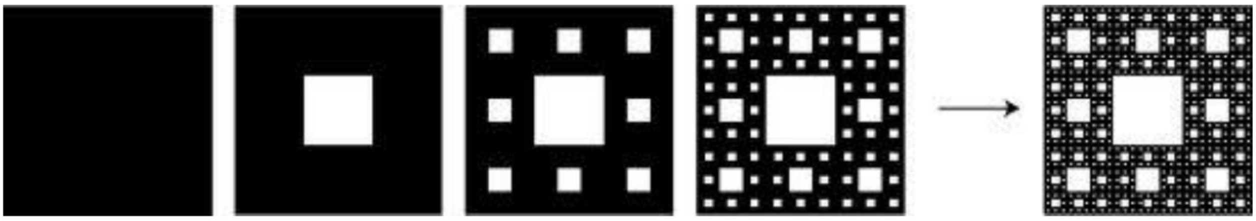
And, of course, the many articles in the Google Drive "Fractals" folder:
https://drive.google.com/a/dragons.asms.net/folderview?id=0B26XdI9aXnUQYWExNUZsTmJ1Z1k&usp=sharing

Sierpinski Carpet:



Let S_n be the nth step of the construction of the Sierpinski Carpet. Suppose that S_0 has area 1.

	S_0	S_1	S_2	S_3
Perimeter of this iteration S_n $4+\sum_{i=1}^{n} 4\left(\frac{8^{i-1}}{3^i}\right)$	$P_0 = 4$	$P_1 = 4+\frac{4}{3}$	$P_2 = 4+\frac{4}{3}+\frac{32}{9}$	$P_3 = P_2+\frac{64(4)}{27}$
Perimeter added to previous iteration to obtain this one $4\left(\frac{8^{n-1}}{3^n}\right)$	N/A	$4/3 = 4\left(\frac{8^0}{3^1}\right)$	$8(4)\left(\frac{1}{9}\right) = 4\left(\frac{8^1}{3^2}\right)$ #of squares edges per sq. edge length	$64(4)\left(\frac{1}{27}\right) = 4\left(\frac{8^2}{3^3}\right)$
Area removed from previous iteration to obtain this iteration $\frac{8^{n-1}}{9^n}$	N/A	$1/9 = \frac{8^0}{9^1}$	$\frac{8}{81} = \frac{8}{9^2}$	$\frac{64}{729} = \frac{8^2}{9^3}$
Total area removed up to this point $\sum_{i=1}^{n} \frac{8^{i-1}}{9^i}$	0	$1/9$	$1/9 + 8/81$	$\frac{1}{9}+\frac{8}{81}+\frac{64}{729}$
Number of squares in this iteration 8^n	$1 = 8^0$	$8 = 8^1$	$64 = 8^2$	$64 \cdot 8 = 512 = 8^3$
Area of a square in this iteration $\frac{1}{9^n}$	$1 = \frac{1}{9^0}$	$1/9 = \frac{1}{9^1}$	$\frac{1}{81} = 1/9^2$	$\frac{1}{27^2} = \frac{1}{729} = \frac{1}{(3^3)^2} = \frac{1}{(3^2)^3}$
Total area of this iteration $\left(\frac{8}{9}\right)^n$	$A_0 = 1$	$A_1 = \frac{8}{9}$	$A_2 = \frac{64}{81}$	$A_3 = \frac{512}{729}$
Area of this iteration divided by area of previous iteration $\frac{A_n}{A_{n-1}} = \frac{8}{9}$	N/A	$\frac{A_1}{A_0} = \frac{8}{9}$	$\frac{A_2}{A_1} = \frac{8}{9}$	$\frac{A_3}{A_2} = 8/9$



S_0 S_1 S_2 S_3

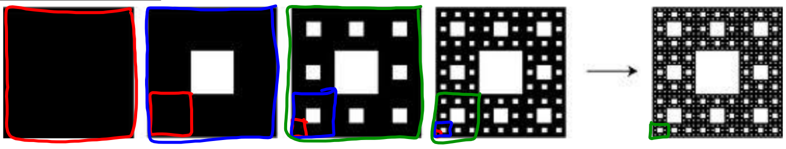
Perimeter of this iteration	S_n $4+\sum_{i=1}^{n} 4\left(\frac{8^{n-1}}{3^n}\right)$
Perimeter added to previous iteration to obtain this one	$4\left(\frac{8^{n-1}}{3^n}\right)$
Area removed from previous iteration to obtain this iteration	$\frac{8^{n-1}}{9^n}$
Total area removed up to this point	$\sum_{i=1}^{n} \frac{8^{i-1}}{9^i} = \sum_{i=0}^{n} \frac{8^i}{9^{i+1}} = \sum_{i=0}^{n} \frac{1}{9}\left(\frac{8}{9}\right)^i$
Number of squares in this iteration	8^n
Area of a square in this iteration	$\frac{1}{9^n}$
Total area of this iteration	$\left(\frac{8}{9}\right)^n \longrightarrow 0$ as $n \to \infty$
Area of this iteration divided by area of previous iteration	$\frac{A_n}{A_{n-1}} = \frac{8}{9}$

$$S_\infty = \frac{1/9}{1-8/9} = \frac{1/9}{1/9} = 1$$

total area removed is 1 !

geometric series w/ common ratio 8/9

Sierpinski Carpet:



S_0 S_1 S_2 S_3

The area of S_n equals $\underline{8/9}$ times the area of S_{n-1}. Therefore the area of S_n is $\underline{(8/9)^n}$.

The area of S_n can also be found as follows: S_n consists of $\underline{8^n}$ squares of area $\underline{1/9^n}$. Therefore area of S_n equals $\underline{(8/9)^n}$

The area removed to obtain S_{n+1} from S_n equals $\underline{8^n/9^{n+1}}$. Thus total area removed can be found as the sum of a series (write down the series and calculate its sum):

$$\sum_{i=0}^{n} \frac{8^i}{9^{i+1}} = \sum_{i=0}^{n} \frac{1}{9}\left(\frac{8}{9}\right)^i = \frac{1/9}{1-8/9} = 1$$

geometric series w/ common ratio 8/3

Find the (total) perimeter of S_n. Explain and show your work.

$$4+\sum_{i=1}^{n} 4\left(\frac{8^{n-1}}{3^n}\right) = 4+\sum_{i=0}^{n} 4\left(\frac{8^n}{3^{n+1}}\right) = 4+\frac{4}{3}\sum_{i=0}^{n}\left(\frac{8}{3}\right)^n \to \infty \text{ as } n\to\infty$$

~~Find the fractal dimension of the Sierpinski Carpet.~~

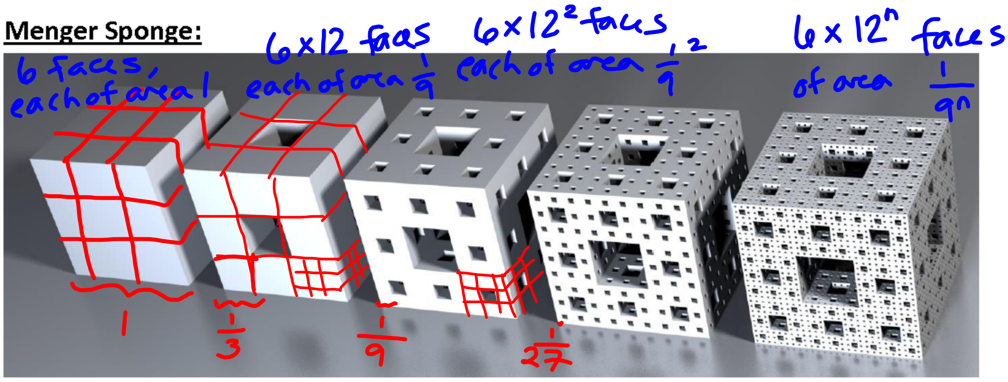
Infinite perimeter, but zero area.

Fractal Dimension:

$S = 3$; $N = 8$

$$d = \frac{\log N}{\log S} = \frac{\log 8}{\log 3} \approx 1.89$$

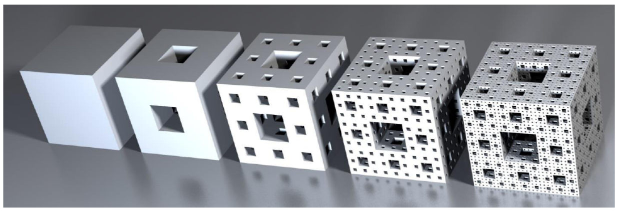
Menger Sponge:



For the **Menger Sponge**, assume that the volume of M_0 is 1.

M_n	M_0	M_1	M_2	M_3
Surface area of this iteration	$A_0 = 6$	$A_1 = 8$	$A_2 =$	$A_3 =$
Volume removed from previous iteration to obtain this iteration $7\left(\frac{20^{n-1}}{27^n}\right)$	N/A	$7\cdot\frac{1}{27}=\frac{7}{27}=7\left(\frac{20^0}{27^1}\right)$	$7\cdot 20\left(\frac{1}{27^2}\right)=7\left(\frac{20^1}{27^2}\right)$	$7\left(\frac{20^2}{27^3}\right)$
Total volume removed up to this point $\sum_{i=1}^{n} 7\left(\frac{20^{i-1}}{27^i}\right)$	0	$\frac{7}{27}$	$\frac{7}{27}+\frac{7(20)}{27^2}$	$\frac{7}{27}+\frac{7(20)}{27^2}+\frac{7(20^2)}{27^3}$
Number of cubes in this iteration 20^n	1	20	20^2	20^3
Volume of a cube in this iteration $\frac{1}{27^n}$	$1=\frac{1}{27^0}$	$\frac{1}{27}=\frac{1}{3^3}=\frac{1}{27^1}$	$\frac{1}{9^3}=\frac{1}{(3^2)^3}=\frac{1}{(3^3)^2}=\frac{1}{27^2}$	$\frac{1}{27^3}=$
Total volume of this iteration $\left(\frac{20}{27}\right)^n$	$V_0 = 1$	$V_1 = \frac{20}{27}$	$V_2 = \left(\frac{20}{27}\right)^2$	$V_3 = \left(\frac{20}{27}\right)^3$
Volume of this iteration divided by volume of previous iteration	N/A	$\frac{V_1}{V_0} = \frac{20}{27}$	$\frac{V_2}{V_1} = \frac{20}{27}$	$\frac{V_3}{V_2} = \frac{20}{27}$

Menger Sponge:



Find the volume of M_n.

$\left(\frac{20}{27}\right)^n$

Find the volume removed to obtain M_{n+1} from M_n.

$7\left(\frac{20^n}{27^{n+1}}\right)$

Compute the total volume removed.

$$\sum_{i=1}^{n} 7\left(\frac{20^{i-1}}{27^i}\right) = \sum_{i=0}^{n} 7\,\frac{20^i}{27^{i+1}} = \frac{7}{27}\sum_{i=0}^{n}\left(\frac{20}{27}\right)^i$$

geometric series w/ common ratio $20/27$

$$S_\infty = \frac{7/27}{1-20/27} = 1$$

~~Find the fractal dimension of the Menger Sponge M.~~

Surface Area

$6\left(\frac{4}{3}\right)^n \longrightarrow \infty$

Fractal Dimension

$S=3$; $N=20$

$$d = \frac{\log 20}{\log 3} \approx 2.73$$