Planar Symmetry & Wallpaper Groups

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There are exactly 17 so-called "wallpaper groups,"
which, in orbifold notation, can be classified as:

1. o
2. xx
3. *x
4. **
5. 632
6. *632
7. 333
8. *333
9. 3*3
10. 442
11. *442
12. 4*2
13. 2222
14. 22x
15. 22*
16. *2222
17. 2*22

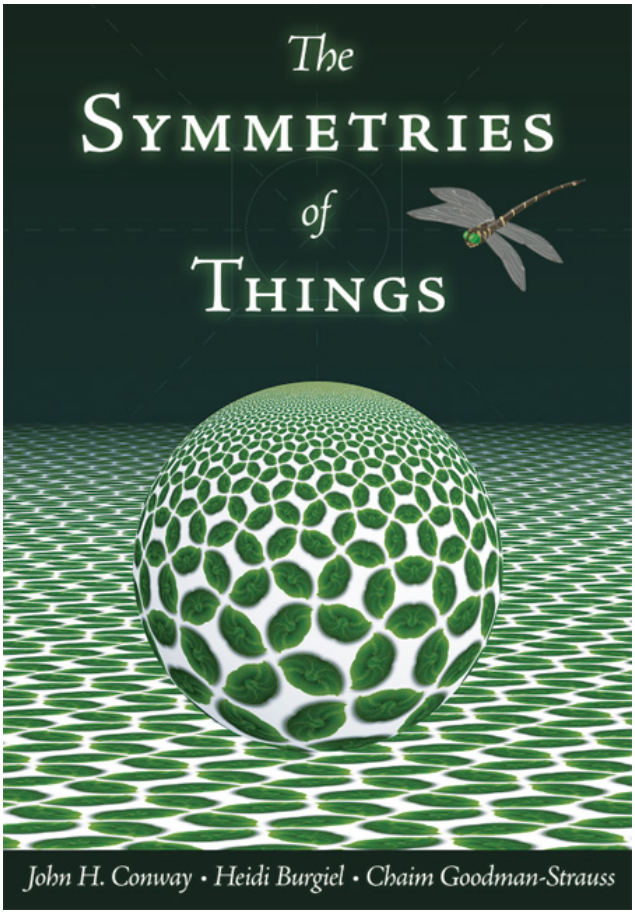
where the notations mean the following:

o is a "wonder ring" or "wandering" and indicates two independent directions of **translation**

x is a "miracle" or **mirrorless glide reflection**

***** is a **mirror line**, and any numbers after a * indicate different points with that many mirror lines passing through those points

numbers before a * or numbers without any other notation indicate points with that number-fold **rotational symmetry**



Symbol	Cost ($)	Symbol	Cost ($)
○	2	∗ or ×	1
2	$\frac{1}{2}$	2	$\frac{1}{4}$
3	$\frac{2}{3}$	3	$\frac{1}{3}$
4	$\frac{3}{4}$	4	$\frac{3}{8}$
5	$\frac{4}{5}$	5	$\frac{2}{5}$
6	$\frac{5}{6}$	6	$\frac{5}{12}$
⋮	⋮	⋮	⋮
N	$\frac{N-1}{N}$	N	$\frac{N-1}{2N}$
∞	1	∞	$\frac{1}{2}$

Table 3.1. Costs of symbols in signatures.

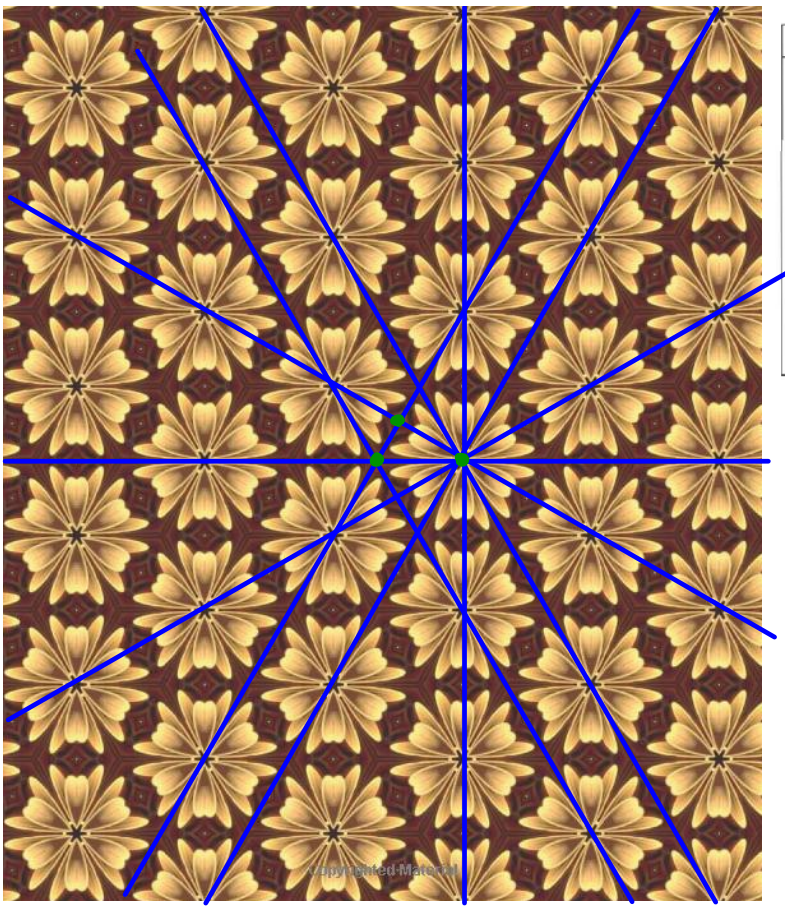


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Table 3.1. Costs of symbols in signatures.

$*632$

$\$1 + \frac{5}{12} + \frac{1}{3} + \frac{1}{4}$

$\frac{12}{12} + \frac{5}{12} + \frac{4}{12} + \frac{3}{12}$

$= \$2$

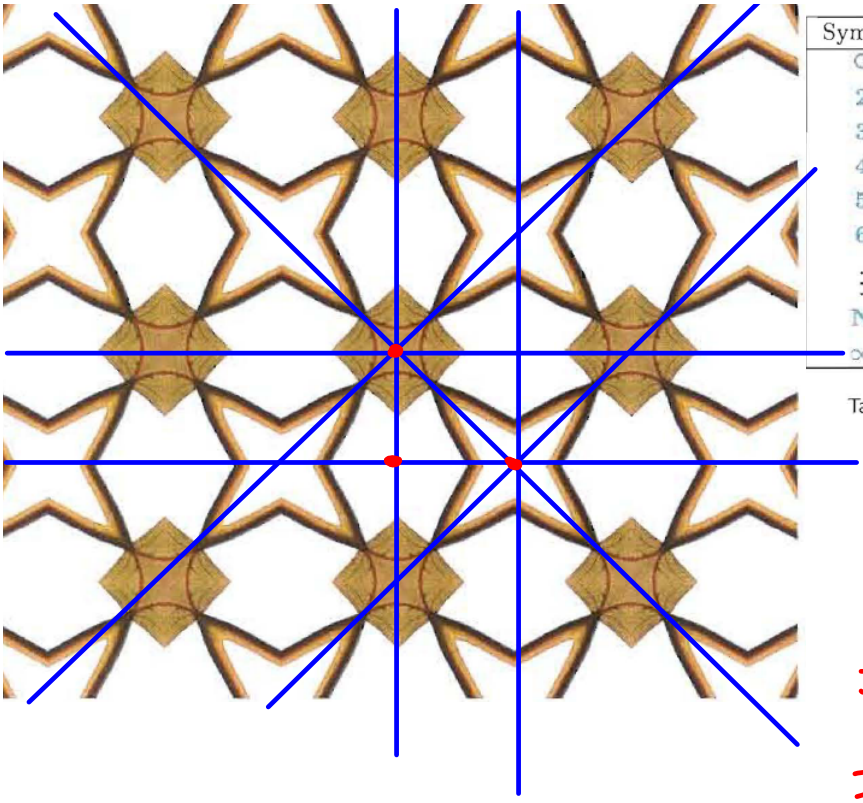

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Table 3.1. Costs of symbols in signatures.

$*442$

$\$1 + \frac{3}{8} + \frac{3}{8} + \frac{1}{4}$

$= 1 + \frac{6}{8} + \frac{2}{8}$

$= \$2$

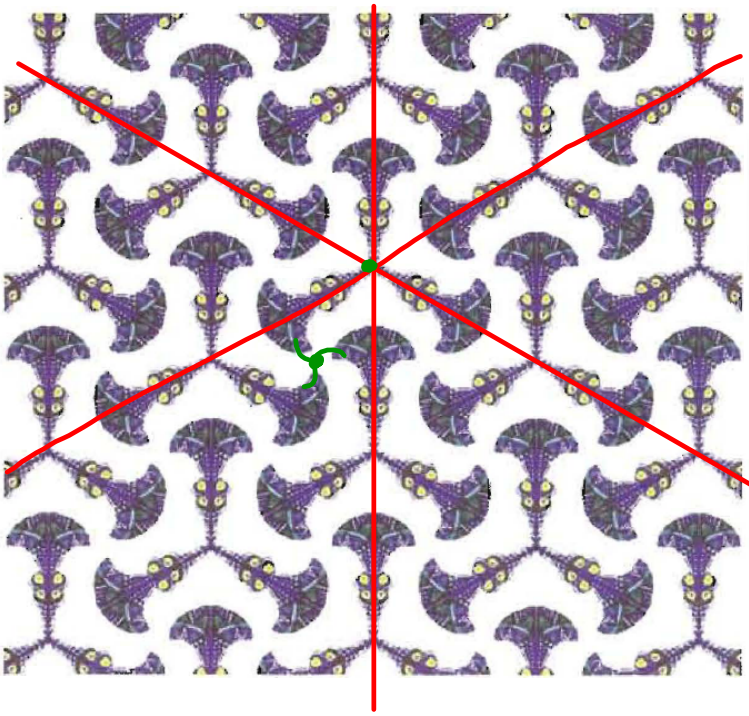

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Table 3.1. Costs of symbols in signatures.

$3*3$

$= \$\frac{2}{3} + 1 + \frac{1}{3}$

$= \$2$

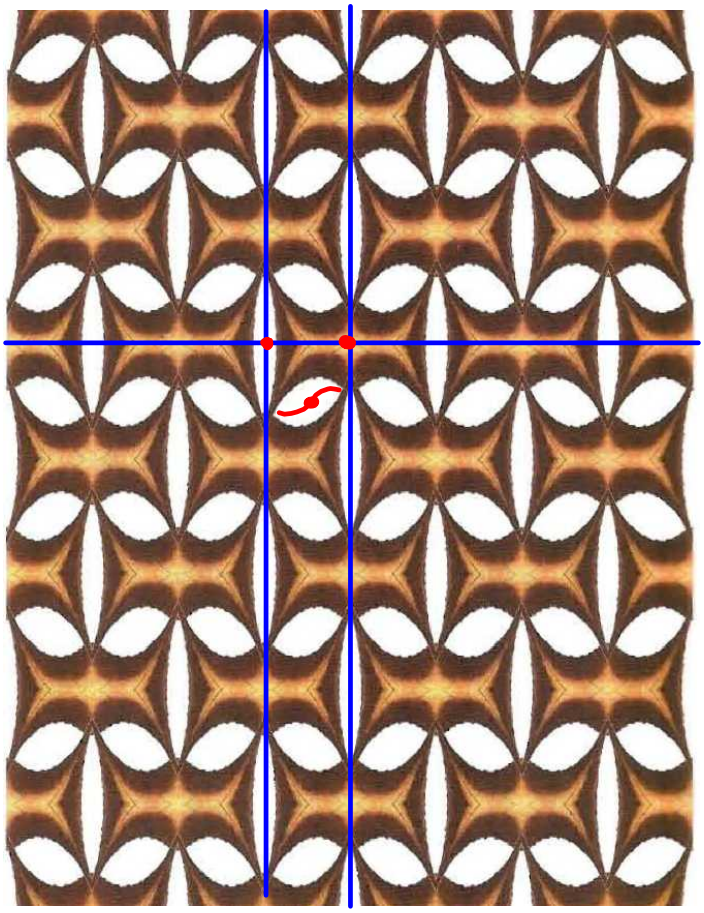

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Table 3.1. Costs of symbols in signatures.

2*22

$= \$ \frac{1}{2} + 1 + \frac{1}{4} + \frac{1}{4}$

$= \$2$

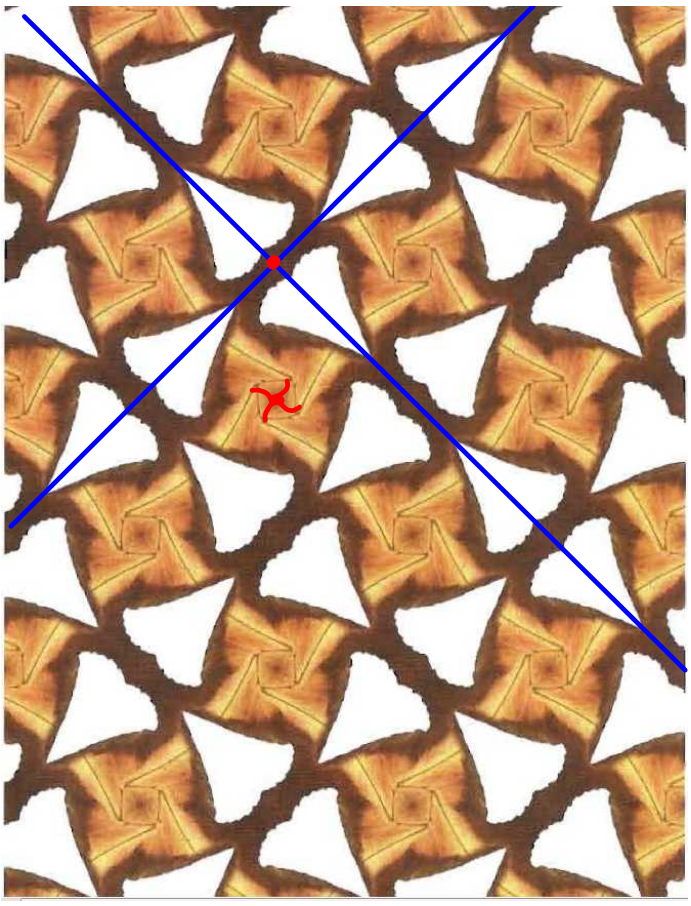

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Table 3.1. Costs of symbols in signatures.

4*2

$= \$ \frac{3}{4} + 1 + \frac{1}{4}$

$= \$2$

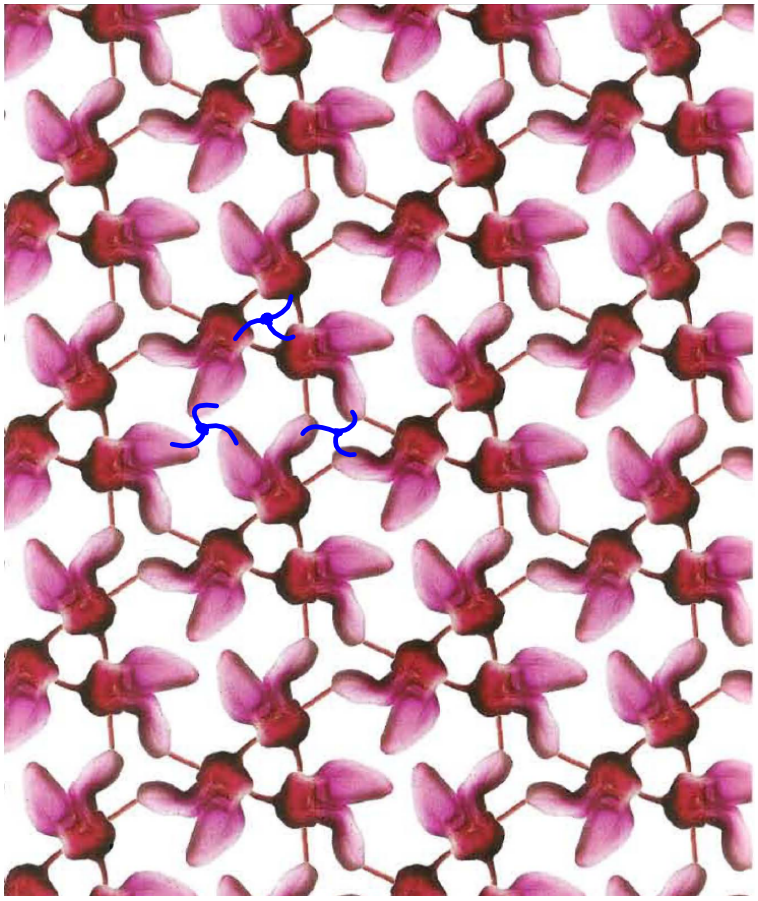

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Table 3.1. Costs of symbols in signatures.

333

$= \$\frac{2}{3} + \frac{2}{3} + \frac{2}{3}$

$= \$2$

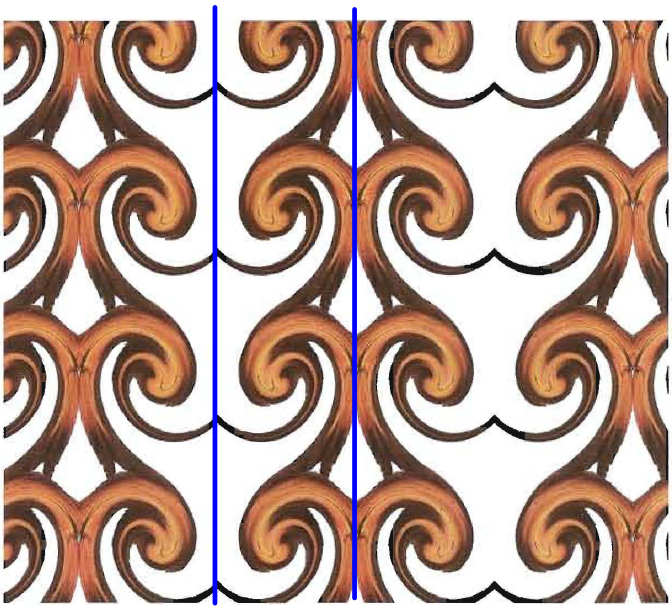

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Table 3.1. Costs of symbols in signatures.

* *

$= \$1 + 1$

$= \$2$

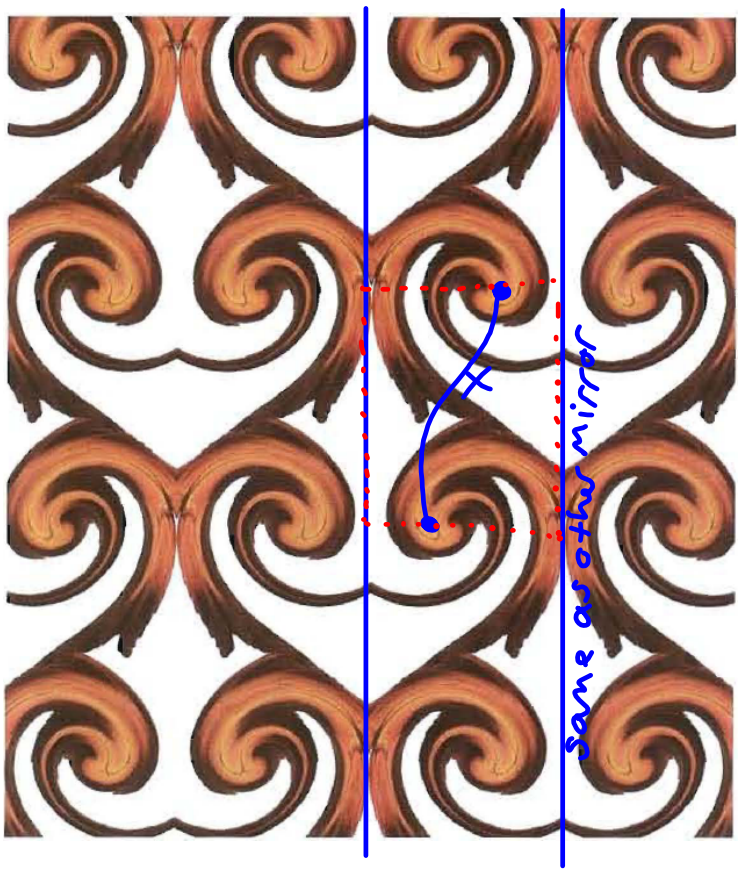

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Table 3.1. Costs of symbols in signatures.

* ×

= $1 + 1

= $2

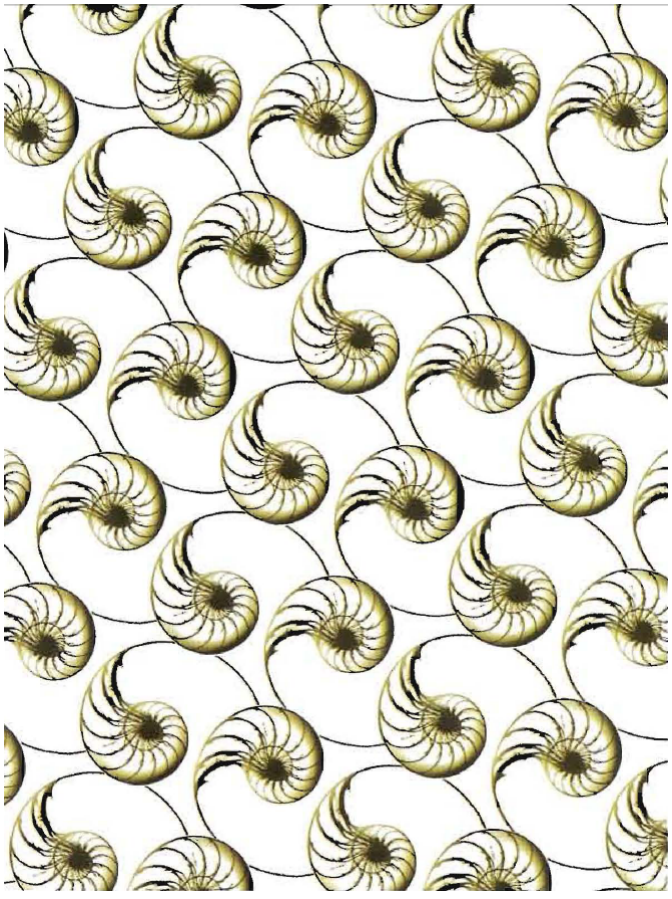

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Table 3.1. Costs of symbols in signatures.