Def Let X and Y be topological spaces and $f: X \to Y$ be a function. f is said to be **continuous** if for every open set V in Y, $f^{-1}(V)$ is open in X.
$f^{-1}(V) = \{x \in X | f(x) \in V\}$

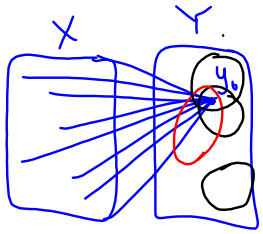
If $\mathcal{B}$ is a basis for Y, then f is continuous if $f^{-1}(B)$ is open for all $B \in \mathcal{B}$.

Examples:

1. $f: X \to Y$, $f(x) = y_0 \in Y$ for all $x \in X$ (constant function)

Let V be an open set in Y.

$$f^{-1}(V) = \begin{cases} X & \text{if } y_0 \in V \\ \emptyset & \text{if } y_0 \notin V \end{cases}$$



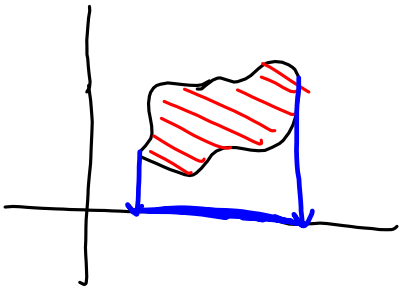
$X, \emptyset$ open in X $\Rightarrow$ f is continuous

2. $\pi_1 : X \times Y \to X$

$\pi_1(x, y) = x$ projection onto first coordinate

Let V be an open set in X.

$\pi_1^{-1}(V) = V \times Y$

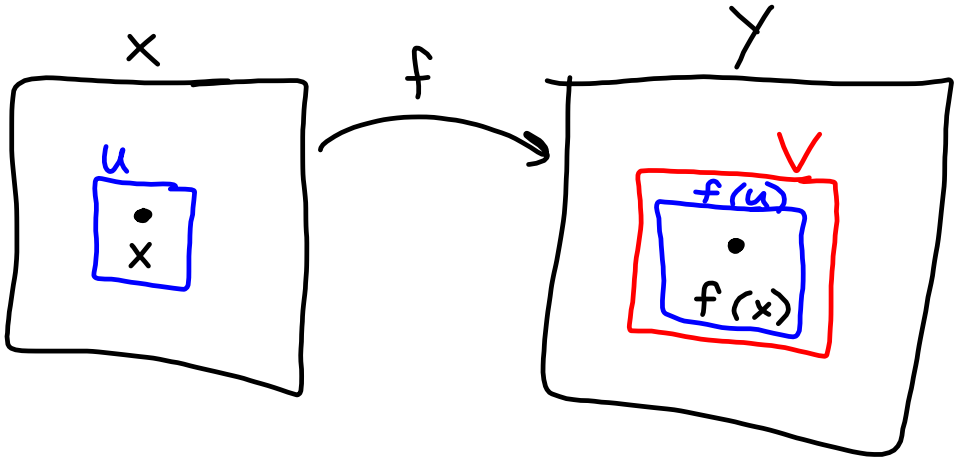


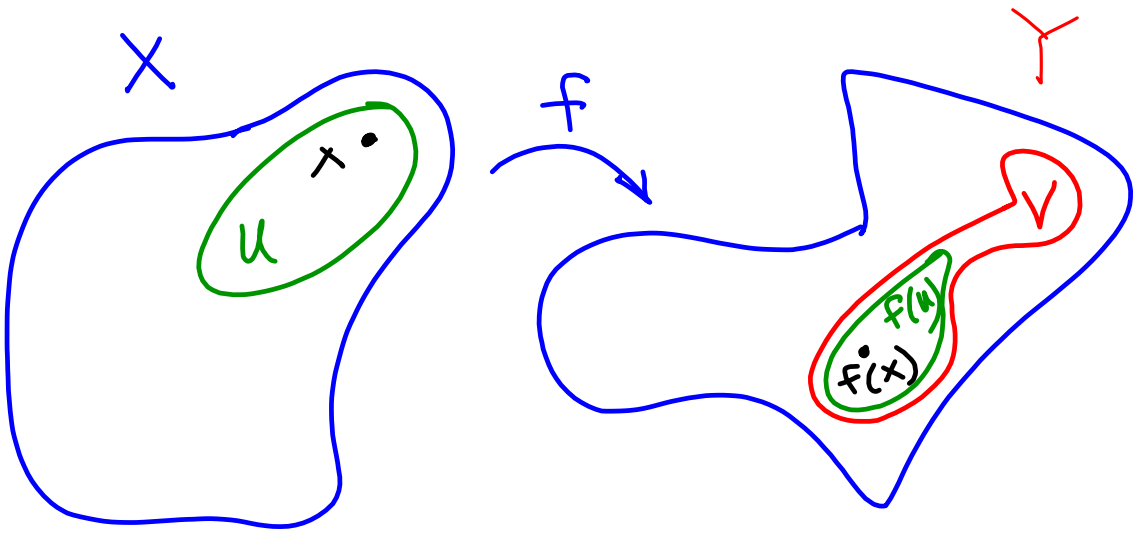
cross-product of open sets is open

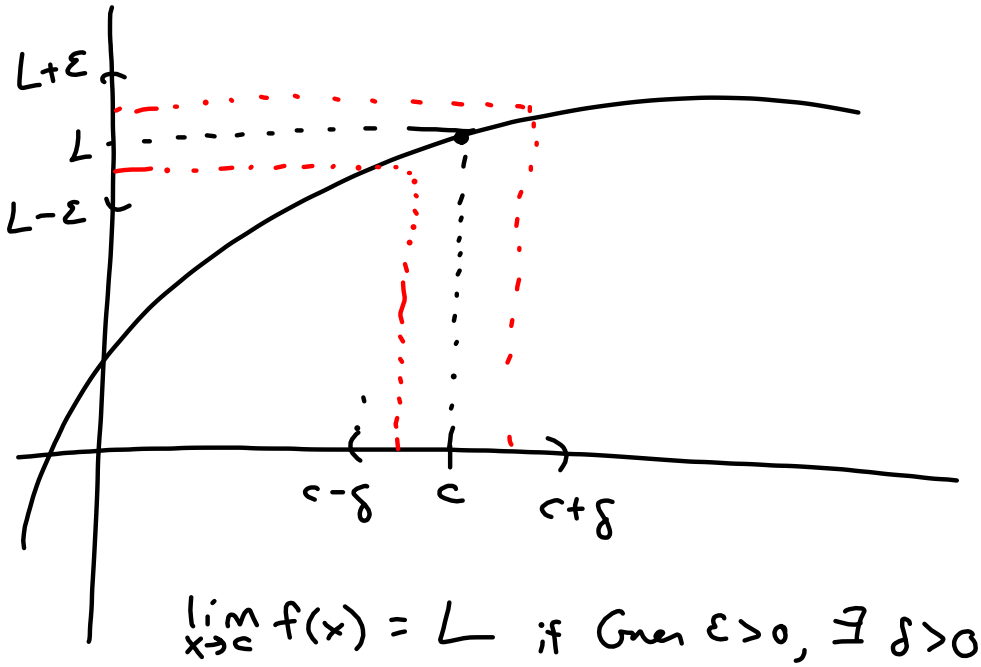
(Genevieve will explain why)

Thm 18.1 Let X and Y be topological spaces and $f: X \to Y$ be a function. The following are equivalent:

1) f is continuous
2) $f^{-1}(B)$ is closed for every closed set $B \subset Y$
3) for each $x \in X$ and each neighborhood V of $f(x)$, there exists a neighborhood U of x such that $f(U) \subset V$.



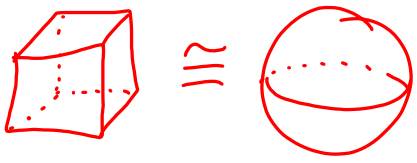




$\lim_{x \to c} f(x) = L$ if Given $\varepsilon > 0$, $\exists\ \delta > 0$ s.t. $|f(x) - L| < \varepsilon$ whenever $|x - c| < \delta$.

f is cts if $f(c) = L$.

Def Let $f: X \to Y$ be a bijection. If both f and f^{-1} are continuous, then f is called a **homeomorphism**.



$\simeq$ "is homeomorphic to"