$\{a_n\}$ is cauchy if given $\varepsilon>0$ $\exists N>0$ s.t. $|a_n-a_m|<\varepsilon$ $\forall n,m\geq N$

Let $\varepsilon>0$ be given. Take $N>\frac{4}{\varepsilon^2}$ s.t. $\frac{1}{\sqrt{n}}<\frac{\varepsilon}{2}$ $\forall n\geq N$

$$|a_n-a_m| = \left|\left(1+\frac{1}{\sqrt{n}}\right)-\left(1+\frac{1}{\sqrt{m}}\right)\right|$$

$$= \left|\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{m}}\right| = \left|\frac{\sqrt{m}-\sqrt{n}}{\sqrt{nm}}\right|$$

$$\left|\frac{\sqrt{m}-\sqrt{n}}{\sqrt{nm}}\right| \leq \left|\frac{\sqrt{m}+\sqrt{n}}{\sqrt{mn}}\right| \leq \frac{1}{\sqrt{m}}+\frac{1}{\sqrt{n}} < \frac{\varepsilon}{2}+\frac{\varepsilon}{2}=\varepsilon$$

For all $m+n>N$

$$\frac{1}{\sqrt{n}}<\frac{\varepsilon}{2}$$

$$\frac{2}{\varepsilon}<\sqrt{n}$$

$$n>\frac{4}{\varepsilon^2}$$

Given: $\{x_n\}, \{y_n\}$ - Cauchy of reals

To show: $\lim_{n\to\infty}(x_n-y_n)$ exists.

$\{x_n\}$ cauchy $\Rightarrow$ Given $\varepsilon>0$ $\exists N_1>0$ s.t. $|x_n-x_m|<\varepsilon$ $\forall n,m\geq N_1$

$\{y_n\}$ cauchy $\Rightarrow$ $\exists N_2>0$ s.t. $|y_n-y_m|<\varepsilon$ $\forall n,m\geq N_2$.

Let $\varepsilon>0$ be given. Take N s.t. $|x_n-x|<\varepsilon/2$ & $|y_n-y|<\varepsilon/2$ $\forall n\geq N$

By the Axiom of Completeness, $x_n\to x$ & $y_n\to y$.

$$|x_n-y_n-(x-y)| = |x_n-x+y-y_n| \leq |x_n-x|+|y-y_n| < \varepsilon/2+\varepsilon/2=\varepsilon.$$