

For the given word, label the edges (with orientation) and vertices of the corresponding polygon.

Determine the number of vertices (# of different capital letters used to label vertices in polygon), edges (# of different lowercase letters in word), and faces (# of words), and use these values to calculate the Euler characteristic.

$$\chi(S) = v - e + f$$

Determine if the surface is orientable or non-orientable.

Does the surface include a Mobius band? If not, it's orientable; if so, it's non-orientable.

Sketch the unassociated edges with connectivity and use this to determine the number of boundary components.

Use the Euler characteristic, orientability, and number of boundary components to calculate the genus of the surface.

Note that the genus is always a nonnegative integer.

If orientable,  $\chi(S) = 2 - 2g - b$ , or, rearranged to solve for  $g$ ,  $g = \frac{2-b-\chi(S)}{2}$

If non-orientable,  $\chi(S) = 2 - g - b$ , or, rearranged to solve for  $g$ ,  $g = 2 - b - \chi(S)$

Use the genus and number of boundary components to describe the number of handles or cross-handles and holes that the surface has, and if possible, give the common name for the surface.

Sketch the surface.

Summary of Variables and Formulas

Notation:

$\chi(S)$  - Euler characteristic of the surface

$b$  - # of boundary components (holes, perforations)

$g$  - genus of the surface (number of handles or crosscaps)

$v$  - # of vertices

$e$  - # of edges

$f$  - # of faces

Formulas:

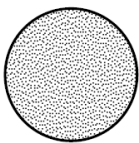
$$\chi(S) = v - e + f \quad (\text{for any surface})$$

$$\chi(S) = 2 - 2g - b \quad (\text{for orientable surfaces})$$

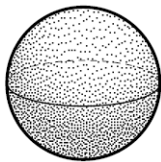
$$\chi(S) = 2 - g - b \quad (\text{for non-orientable surfaces})$$

Common surfaces:

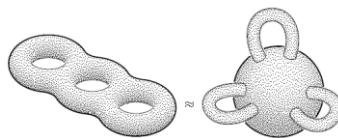
Disk



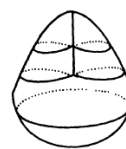
Sphere



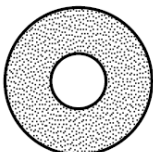
2-torus, 3-torus, ... K-torus



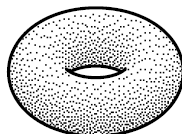
Projective plane



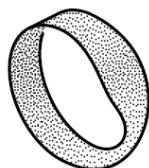
Annulus



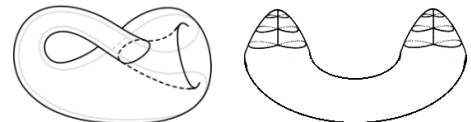
Torus



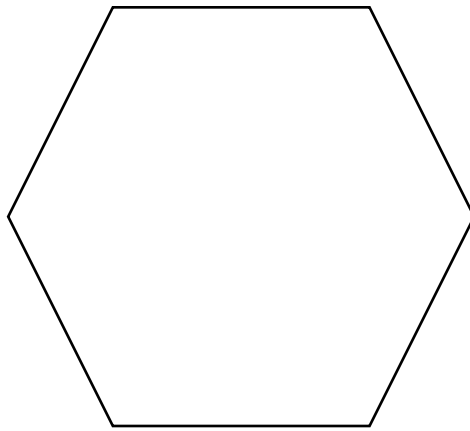
Mobius band



Klein bottle



1.  $bre^{-1}wer^{-1}$



# Vertices:

# Edges:

# Faces:

Euler characteristic:

Orientable / Non-orientable

Sketch of boundary components:

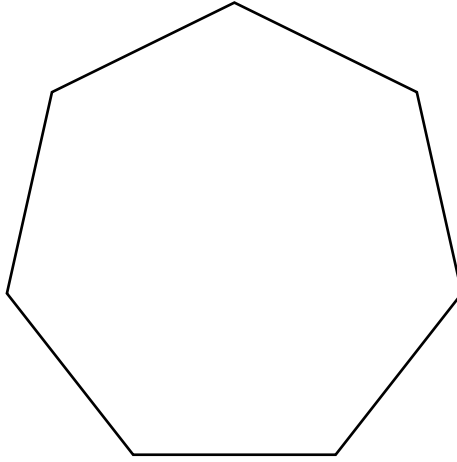
Genus:

This is a(n) **orientable / non-orientable** surface of **genus** \_\_\_\_\_ with \_\_\_\_\_ **boundary components**. Hence, it is topologically equivalent to a sphere with \_\_\_\_\_ **handles / cross-caps** and \_\_\_\_\_ **holes**.

Common name:

Sketch of surface:

2.  $rac^{-1}ecar^{-1}$



# Vertices:

# Edges:

# Faces:

Euler characteristic:

Orientable / Non-orientable

Sketch of boundary components:

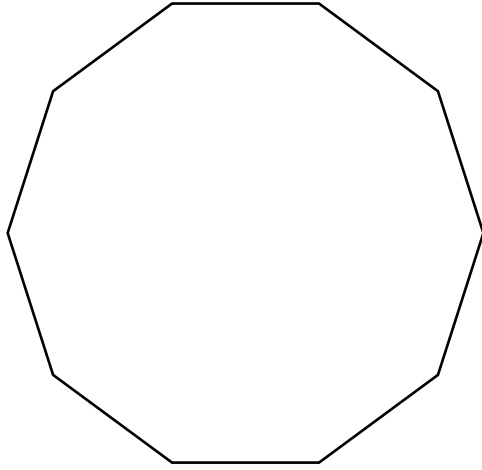
Genus:

This is a(n) **orientable / non-orientable** surface of **genus** \_\_\_\_\_ with \_\_\_\_\_ **boundary components**. Hence, it is topologically equivalent to a sphere with \_\_\_\_\_ **handles / cross-caps** and \_\_\_\_\_ **holes**.

Common name:

Sketch of surface:

3.  $in^{-1}te^{-1}s^{-1}t^{-1}i^{-1}nes$



# Vertices:

# Edges:

# Faces:

Euler characteristic:

Orientable / Non-orientable

Sketch of boundary components:

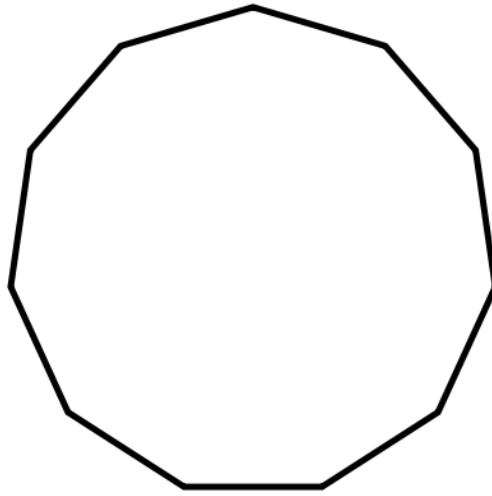
Genus:

This is a(n) **orientable / non-orientable** surface of **genus** \_\_\_\_\_ with \_\_\_\_\_ **boundary components**. Hence, it is topologically equivalent to a sphere with \_\_\_\_\_ **handles / cross-caps** and \_\_\_\_\_ **holes**.

Common name:

Sketch of surface:

4.  $u^{-1}nd^{-1}ergr^{-1}ou^{-1}nd$



# Vertices:

# Edges:

# Faces:

Euler characteristic:

Orientable / Non-orientable

Sketch of boundary components:

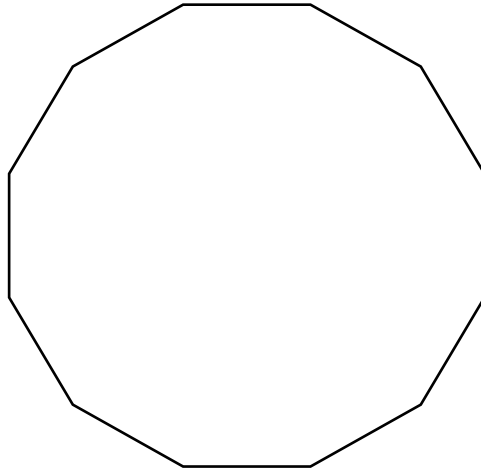
Genus:

This is a(n) **orientable / non-orientable** surface of **genus** \_\_\_\_\_ with \_\_\_\_\_ **boundary components**. Hence, it is topologically equivalent to a sphere with \_\_\_\_\_ **handles / cross-caps** and \_\_\_\_\_ **holes**.

Common name:

Sketch of surface:

5.  $me^{-1}dabb^{-1}am^{-1}ed^{-1}oo^{-1}$



# Vertices:

# Edges:

# Faces:

Euler characteristic:

Orientable / Non-orientable

Sketch of boundary components:

Genus:

This is a(n) **orientable / non-orientable** surface of **genus** \_\_\_\_\_ with \_\_\_\_\_ **boundary components**. Hence, it is topologically equivalent to a sphere with \_\_\_\_\_ **handles / cross-caps** and \_\_\_\_\_ **holes**.

Common name:

Sketch of surface: