Trigonometry section 5.4 – Radians, Arc Length, Linear Speed, and Angular Speed

Variables:

$s = distance\ traveled$ or $arc\ length\ (inches, kilometers, etc)$

$t = time\ (seconds, minutes, hours, etc)$

$\theta = amount\ of\ rotation$ or $included\ angle\ (degrees, radians, rotations, revolutions, etc)$

$r = radius\ or\ distance\ from\ the\ center\ of\ rotation\ (centimeters, inches, etc)$

$v = linear\ speed = \dfrac{distance}{time}$

$\omega = angular\ speed = \dfrac{amount\ of\ rotation}{time}$

Formulas:

$s = r\theta\ ,\qquad v = \dfrac{s}{t}\ ,\qquad \omega = \dfrac{\theta}{t}\ ,\qquad v = r\omega$

Note: We derive $v = r\omega$ by combing $v = \frac{s}{t}$ with $s = r\theta$ to get $v = \frac{s}{t} = \frac{r\theta}{t} = r \cdot \frac{\theta}{t} = r\omega$

Dimensional analysis conversion factors:

$\dfrac{5280\ ft}{1\ mi}\ ,\dfrac{12\ in}{1\ ft}\ ,\dfrac{2\pi}{1\ rev}\ ,\dfrac{\pi}{180°}\ ,\dfrac{60\ min}{1\ hr}\ ,\dfrac{60\ sec}{1\ min}\ ,and\ their\ reciprocals$

Steps to Solve:

1. Identify what is given and what you are trying to find; identify all variables and associated units.
2. Determine which equation relates the known and unknown variables.
3. Rearrange the equation to solve for the unknown variable.
4. Plug in known quantities with units into equation.
5. Multiply by a series of dimensional analysis conversion factors until you arrive at the appropriate units for your answer.

Example Problems:

1. A wheel with a 15-inch diameter rotates at a rate of 6 radians per second. What is the linear speed of a point on its rim in feet per minute?

Solution: $r = \frac{15}{2} in\ ,\ \omega = 6\frac{rad}{s}\ ,\ v = ?\ ft/min$ Equation: $v = r\omega$

$$v = r\omega = \frac{15\ in}{2} \cdot \frac{6\ rad}{1\ s} \cdot \frac{1\ ft}{12\ in} \cdot \frac{60\ s}{1\ min} = \boxed{225\ ft/min}$$

2. An earth satellite in circular orbit 1200 km high makes one complete revolution every 90 minutes. What is its linear speed in km/min, given that the earth's radius is 6400 km?

Solution: $r = 6400 + 1200 = 7600\ km\ ,\ \omega = \frac{1\ rev}{90\ min}\ ,\ v = ?\ km/min$ Equation: $v = r\omega$

$$v = r\omega = \frac{7600km}{1} \cdot \frac{1\ rev}{90\ min} \cdot \frac{2\pi}{1\ rev} = \frac{760 \cdot 2\pi\ km}{9\ min} = \boxed{\frac{1520\pi}{9} km/min}$$

3. Through how many radians does the minute hand of a clock rotate from 12:45pm to 1:25pm?

Solution: $\theta = ?,\ t = 40\ min,\ \omega = \frac{2\pi}{1\ hr}$ Equation: $\omega = \frac{\theta}{t}$

$$\theta = \omega t = \frac{2\pi}{1\ hr} \cdot \frac{40\ min}{1} \cdot \frac{1\ hr}{60\ min} = \boxed{\frac{4\pi}{3}}$$

4. A car travels at 60 miles per hour. Its wheels have a 24-inch diameter. What is the angular speed of a point on the rim of a wheel in revolutions per minute?

Solution: $v = \frac{60\ mi}{1\ hr}, = 12\ in, \omega = ? \frac{rev}{min}$ Equation relating these variables: $v = r\omega$

$$\omega = \frac{v}{r} = v \cdot \frac{1}{r} = \frac{60\ mi}{1\ hr} \cdot \frac{1}{12\ in} \cdot \frac{1\ hr}{60\ min} \cdot \frac{5280\ ft}{1\ mi} \cdot \frac{12\ in}{1\ ft} \cdot \frac{1\ rev}{2\pi} = \boxed{\frac{2640}{\pi} \frac{rev}{min}}$$

Homework Problems to start working in class:

Do <u>not</u> use a calculator to solve these problems. Numbers should cancel nicely. Follow the above examples. Typically, if your units end up correct you are doing the problems correctly, but be careful! Also, don't forget about your π's – if the problem has a π in the denominator that doesn't cancel out, your answer should also have a π in the denominator.

1. A car travels 3 miles. Its tires make 2640 revolutions. What is the radius of a tire in inches?

2. A satellite 290 miles above Mars' surface makes one revolution every 2 hours. What is its linear speed in miles per hour, given that the radius of Mars is 2110 miles?

3. A pulley has a 48-inch diameter, and moves a belt at a rate of 8 miles per hour. What is the angular speed of a point on the edge of the pulley in revolutions per minute?

4. A circle has a radius of 3 feet. What is the measure, in degrees, of an angle that subtends an arc of 4 inches?

5. The angle of depression to the bottom of a slide is 60 degrees. If a child slides down at a rate of 5 feet per second and it takes 2 seconds for the child to reach the bottom, what is the vertical height of the slide, in feet?

Additional homework from textbook:

<u>5.4b</u> #61-79 odd

Note that the answers given in the book are rounded, and the problems do not give you nice numbers that cancel neatly like the 5 problems above. Work the problems in the same way as the given examples and cancel as much as you can, but to check your answers, plug what you got into a calculator to check.