Trigonometry Test #3 Name____

Brewer Fall 2009

Directions: Read each question carefully. Show all of your work. Except in the case of proving identities, **circle your exact, simplified answer**. Each problem is worth 10 points.

Half-angle identities:

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}\ , \qquad \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}\ , \qquad \tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$$

Sum and difference identities:

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b\ , \quad \cos(a \pm b) = \cos a \cos b \mp \sin a \sin b\ ,$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

1. Use the half-angle identity to evaluate $\tan\frac{7\pi}{12}$ exactly.

2. Find the exact value of $\cos 212° \cos 122° + \sin 212° \sin 122°$.

3. Find $\sin 2\theta, \cos 2\theta, and \tan 2\theta$ given that $\cos\theta = \frac{12}{13}$ and θ is in Quadrant IV.

4. Given $\sin\alpha = \frac{12}{13}$, α is in Quadrant II, $\cos\beta = -\frac{4}{5}$, and β is in Quadrant III, find $\sin(\alpha + \beta)$.

5. Find $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ exactly in radians.

6. Evaluate $\cos\left(\csc^{-1}\frac{2}{\sqrt{3}}\right)$

7. Find all solutions (in radians) in the interval $0 \leq x < 2\pi$. $\sin^2 x - \frac{1}{4} = 0$

8. Find all solutions (in radians) in the interval $0 \le x < 2\pi$. $2\sin^3 x = \sin x$

9. Prove the identity.

$$\frac{1+\cos^2 x}{\sin^2 x} = 2\csc^2 x - 1$$

10. Prove the identity. $\csc x - \cos x \cot x = \sin x$

Bonus (10 points): Find all solutions (in radians) in the interval $0 \leq x < 2\pi$.

$$\sin 3x + \sin x - \sin 2x = 0$$